HW6 - Problems

Andrea Brose, Math 167/1, F05

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AP(x) Let $P \subset \mathbb{R}^n$ and $Q \subset \mathbb{R}^m$ be convex sets. Show that

$$P \times Q = \{(\boldsymbol{p}, \boldsymbol{q}) \,:\, \boldsymbol{p} \in P,\, \boldsymbol{q} \in Q\} \subset \mathbb{R}^{n+m}$$

is a convex set.

AP(xi) Consider

$$\begin{aligned} f \,:\, [0,1] &\to \mathbb{R} \\ x &\mapsto \begin{cases} \frac{1}{x} & \text{if } 0 < x \leq 1 \\ 13 & \text{if } x = 0 \end{cases} \end{aligned}$$

(a) Define the correspondence

$$\begin{aligned} R_f \,:\, [0,1] &\rightrightarrows \mathbb{R} \\ x &\mapsto R_f(x) = \{f(x)\}\,, \end{aligned}$$

where f is as defined above. Let Γ_{R_f} be the graph of the correspondence R_f.

i. Is Γ_{R_f} convex set?
ii. Is Γ_{R_f} closed set?

Argue carefully using a picture, i.e. you do not need to provide a rigorous prove, but may if you want.

(b) Define the correspondence

$$\begin{aligned} R \,:\, [0,1] &\rightrightarrows \mathbb{R} \\ x &\mapsto R(x) = \{y \,:\, 0 \leq y \leq f(x)\}\,, \end{aligned}$$

where f is as defined above. Let Γ_R be the graph of the correspondence R.

i. Is Γ_R convex set? How about $R(x)$ for each $x \in [0,1]$?
ii. Is Γ_R closed? How about $R(x)$ for each $x \in [0,1]$?

Argue carefully using a picture, i.e. you do not need to provide a rigorous prove, but may if you want.

AP(xii) Show that $\lim_{n\to\infty} \frac{1}{n} \neq 1$.

AP(xiii) Prove the following proposition from lecture on the 31st of October:

Proposition 9 Let $X \subseteq \mathbb{R}^n$, then every $x \in X$ is a limit point of X.

AP(xiv) Consider the following correspondences R and check whether they are (upper hemi-)continuous[1] or not:

(a) $R_f : \mathbb{R} \rightrightarrows \mathbb{R}$, where $f(x) = x^2$

(b) $R_f : \mathbb{R} \rightrightarrows \mathbb{R}$, where $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$

(c) $R : \mathbb{R} \rightrightarrows \mathbb{R}$, $R(x) = \begin{cases} [-1, 1] & \text{if } x \geq 0 \\ \{0\} & \text{if } x < 0 \end{cases}$

(d) $R : \mathbb{R} \rightrightarrows \mathbb{R}$, $R(x) = \begin{cases} [-1, 1] & \text{if } x > 0 \\ \{0\} & \text{if } x \leq 0 \end{cases}$

AP(xv) Prove the "only if" part following proposition from lecture on the 4th of November:

Proposition 11 A correspondence $R : X \rightrightarrows Y$ is (upper hemi-)continuous if and only if the graph Γ_R of R is closed subset of $X \times Y$.

i.e. prove Γ_R is closed implies R is continuous.

[1]See definition 45 on page 53 of notes from 31.10.2005.