HW10 - Additional Problems

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AP(xxiv) Consider a two-person strategic game whose payoff matrix is given by:

$$M = \begin{bmatrix} (-1,-1) & (1,3) & (3,0) \\ (1,0) & (0,1) & (0,3) \end{bmatrix}.$$

(a) Sketch the feasible payoff region X if players can use coordinated strategies and free disposal is allowed (p. 71 and 72 of notes from 16.11.2005).

(b) Mark the pareto-efficient points of X.

(c) For $\boldsymbol{d}_1 = (0,1)$, mark the bargaining set of X.

(d) For $\boldsymbol{d}_2 = (1,-1)$, mark the bargaining set of X.

(e) Find the generalized bargaining solution to $(X, \boldsymbol{d}_1)$ for the bargaining powers $\alpha = \frac{1}{3}$, $\beta = \frac{2}{3}$.

(f) Find the generalized bargaining solution to $(X, \boldsymbol{d}_2)$ for the bargaining powers $\alpha = \frac{1}{3}$, $\beta = \frac{2}{3}$.

(g) For your answers in (e) and (f) above, write the contracts which yield the respective payoffs of your solution.

AP(xxv) Using "calculus", find the generalized bargaining solution to $(X, \boldsymbol{d})$ for the bargaining powers $\alpha = \frac{3}{5}$ and $\beta = \frac{2}{5}$, where X is the feasible payoff region with pareto efficient points satisfying $x_1^2 + x_2^2 = 1$ for x_1, $x_2 \geq 0$, free disposal allowed, and $\boldsymbol{d} = (0,0)$.

AP(xxvi) Consider the NBP $(X, \boldsymbol{d})$ studied in class on 21.11. and given in the notes in example 47 on page 79. Denoting player's utilities by u and v, define new utilities u' and v' via the following affine transformation τ:

$$\begin{aligned} \tau : \mathbb{R}^2 &\rightarrow \mathbb{R}^2 \\ (u,v) &\mapsto \tau(u,v) = (u-2, 9v) \end{aligned}$$

Let $(X', \boldsymbol{d}')$ be Nash bargaining problem defined by $X' = \tau X$, and $\boldsymbol{d}' = \tau \boldsymbol{d}$.

(a) Let $\alpha = \frac{1}{15}$ and $\beta = \frac{15}{16}$. Find the generalized bargaining solution $\boldsymbol{s}' = G(X', \boldsymbol{d}')$ to $(X', \boldsymbol{d}')$ for bargaining powers α and β two ways:

(i) by using the answer $G(X, \boldsymbol{d}) = \boldsymbol{s}$ to example 47 on page 73 of notes to find $\tau G(X, \boldsymbol{d})$ and

(ii) by computing $G(X', \boldsymbol{d}')$ directly.

(b) Repeat the above for $\alpha = \frac{1}{2} = \beta$, only that you still need to do the calculation for the solution $G(X, \boldsymbol{d}) = \boldsymbol{s}$ for (i) yourself.