

Math 120AB: Differential Geometry

- [Math 120A: General Course Outline & Catalog Description](#)
- [Math 120B: General Course Outline & Catalog Description](#)

Differential geometry can be viewed as the study of space and curvature. The course depends heavily upon calculus, it uses the tools of linear algebra, and it develops geometric insight. As such it is a good course for students who want to strengthen their understanding of the core mathematics curriculum.

Differential geometry is a crucial tool in modern physics. The idea of curved space is at the foundation of Einstein's theory of gravitation (general relativity). Several more recent developments in physics, as Yang-Mills theory and string theory, involve differential geometry.

The courses 120A and 120B deal with differential geometry in a special context, curves and surfaces in 3-space, which has a firm intuitive basis, and for which some remarkable and striking theorems are available.

The course begins with curves in the plane and in 3-space, which already have some interesting geometric features. Curvature and torsion measure how curves bend and twist. There are some beautiful theorems that if a curve in 3-space forms a closed loop, it has to bend at least a certain amount, and if it forms a knot, it has to bend at least a larger certain amount. Another beautiful theorem is the celebrated isoperimetric theorem, that among all closed curves of a fixed length, the circle encloses the largest area.

There are several notions of curvature for surfaces in 3-space. Mean curvature shows up in the problem of determining the surface of the smallest area with a fixed prescribed boundary. (The solution can be illustrated with soap bubbles.) Gaussian curvature shows up in the problem of determining which surfaces can be represented by a flat map.

Another problem treated in the course is how to determine the shortest route on a surface between two points. In the plane the shortest path is a straight line, and on a sphere the shortest path is an arc of a great circle.

The theorem of high-school geometry that the sum of the angles of a triangle is 180 degrees turns out to have a very beautiful generalization to a triangle on any surface (as a spherical triangle). The generalization is the Gauss-Bonnet theorem, which is one of the high-points of undergraduate mathematics. The theorem provides an identity with a sum of angles and a correction term that takes into account how curved the sides of the triangle are and how much the surface is curved inside the triangle. One of the remarkable features of the Gauss-Bonnet theorem is that it asserts the equality of two quantities, one of which comes from differential geometry and the other of which comes from topology.

Math 120AB is highly recommended for mathematics students who want to go on to graduate school. Math 120A is offered once each year. It is followed occasionally by an offering of Math 120B. Course 120A will be offered Fall Quarter 1999. Course 120B will be offered Winter Quarter 2000.