Lecture 1: Lattices in High Dimension

Abstract: A lattice is just a grid in Euclidean space; the subgroup generated by \( n \) linearly independent vectors in \( \mathbb{R}^n \) (e.g. the integer lattice \( \mathbb{Z}^n \)). The problem of understanding the short vectors in a lattice arises in many settings, e.g. in the construction of dense sphere packings and in proposed lattice-based cryptosystems. I will discuss this circle of ideas, and try to convey a picture of what a typical high-dimensional lattice looks like - as a warning, it looks nothing like \( \mathbb{Z}^n \).

Lecture 2 and 3: Hidden Symmetries in the Cohomology of Arithmetic Groups

The overall goal of Lectures 2 and 3 is to explain a program that (conjecturally) connects the motivic cohomology of certain algebraic varieties, to the cohomology of arithmetic groups. Despite the impression you may get from this sentence, lecture 2 will be accessible at the level of a colloquium: absolutely no familiarity with arithmetic groups or motivic cohomology is needed.

2: Arithmetic groups and their cohomology. The basic example of an arithmetic group is the group \( SL_n(\mathbb{Z}) \) of invertible \( n \times n \) integral matrices. I will explain why its group cohomology is interesting, and how one can analyze it using Hodge theory.

3: I will explain the "hidden symmetries" of the cohomology of an arithmetic group, mentioned in the title. These are extra endomorphisms of the cohomology, and conjecturally they will be indexed by the motivic cohomology of certain algebraic varieties. I will try to focus on one striking piece of evidence for this picture, which comes from the theory of analytic torsion on Riemannian manifolds.