HOMEWORK 2

Due on Wednesday, March 10th, in class.

Exercise 1. Let $d = 3$ and $0 < p < 1$. Prove that the initial value problem

$$u_{tt} - \Delta u + |u|^p u = 0$$
$$u(0) = u_0, \quad u_t(0) = u_1$$

is globally wellposed for initial data $(u_0, u_1) \in H^1_x \times L^2_x$.

Exercise 2. (Persistence of regularity) Fix $d = 3$ and consider the initial value-problem

$$u_{tt} - \Delta u \pm |u|^4 u = 0$$
$$u(0) = u_0,\ u_t(0) = u_1$$

with initial data $(u_0, u_1) \in \dot{H}^1_x \times L^2_x$. Assume that the solution satisfies

$$\|u\|_{L^8_{t,x}(I\times\mathbb{R}^3)} \leq M$$

for some positive constant M and some compact time interval I. Suppose that, in addition, the initial data satisfies $(u_0, u_1) \in \dot{H}^k_x \times \dot{H}^{k-1}_x$ for some integer $k \geq 2$. Prove that (u, u_t) is bounded in $\dot{H}^k_x \times \dot{H}^{k-1}_x$ uniformly for $t \in I$.