Homework 1 for Math 215A Commutative Algebra

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Due: Monday, October 8, 2018

Rings are understood to be commutative, unless stated otherwise.

(1) Let R be an algebra over a field k. (That is, R is a ring with a given ring homomorphism $k \to R$.) If R is a domain and R has finite dimension as a k-vector space, then R is a field.

(2) Let k be a field. Show that $k[x]$ and $k[x, x^{-1}]$ are not isomorphic as k-algebras. (Here $k[x, x^{-1}]$ can be defined as the ring of Laurent polynomials $a_{-n}x^{-n} + \cdots + a_n x^n$, where $n \geq 0$ and $a_i \in k$.)

(1) Let k be a field. Show how to view the ring $k[x, y]/(x - 1, x^2 + y^2 - 1)$ as the quotient ring of $k[y]$ by a certain ideal. Deduce that the ideal $(x - 1, x^2 + y^2 - 1)$ is not prime, and compute its radical.