Homework 9 for Math 215A Commutative Algebra

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Due: Wednesday, November 28, 2012

Rings are understood to be commutative, unless stated otherwise.

(1) Let R be a local ring with maximal ideal $\mathfrak{m}$ such that $\mathfrak{m} = \mathfrak{m}^2$. If R is noetherian, show that R is a field. Give an example to show that this can fail for R not noetherian.

(2) Let R be an algebra of finite type over a field. Show that R is a Jacobson ring, which means that every prime ideal in R is an intersection of maximal ideals. Which prime ideals in R can be written as a finite intersection of maximal ideals?

(3) Which of the following rings are Dedekind domains? Justify your answer. $\mathbf{C}[x, y]/(x^4 + xy - 1)$, $\mathbf{Z}[x, y]/(x^4 + xy - 1)$, $\mathbf{F}_2[x, y]/(x^4 - y^3)$.

(4) Let k be a field. Compute $\dim(k[x, y, z]/(x^2 + y^2 + z^2 - 1, xy - z^3))$.