Homework 1 for Math 215A Commutative Algebra

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Rings are understood to be commutative, unless stated otherwise.

(1) Let R be an algebra over a field k. (That is, R is a ring with a given ring homomorphism $k \to R$.) If R is a domain and R has finite dimension as a k-vector space, then R is a field.

(2) Let k be a field. Show that $k[x]$ and $k[x, x^{-1}]$ are not isomorphic as k-algebras. (Here $k[x, x^{-1}]$ can be defined as the ring of Laurent polynomials $a_{-n}x^{-n} + \cdots + a_n x^n$, where $n \geq 0$ and $a_i \in k$.)