Math 131BH Honors Analysis. Sample midterm 2

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This is a sample midterm 2 (from the exams from Winter 2010). It may not exactly match this year's material. The exam on Monday, February 27 will focus on the material from homeworks 3 to 5, that is, chapters 7 and 8 in Rudin.

(1) Does the series

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots$$

converge uniformly on $[0,1)$? Explain.

(2) Decide whether each statement below is true or false. If true, give a proof; if false, give a counterexample, or otherwise disprove it.

(a) If the power series $\sum_{n=0}^{\infty} c_n x^n$ has radius of convergence 1, then $\sum_{n=0}^{\infty} c_n$ converges.

(b) The space $C([0,1])$ of continuous functions on $[0,1]$ with the L^2 norm $||\cdot||_2$ is complete.

(c) The function

$$f(x) = \sum_{n=1}^{\infty} \frac{\sin nx}{n^{5/2}}$$

has a continuous derivative.

(3) If $\sum_{n\in\mathbf{Z}} a_n e^{inx}$ is the Fourier series for the function $f(x) = x$ on $[-\pi,\pi]$, show that $\sum_{n\in\mathbf{Z}} |a_n| = \infty$. (Hint: if $\sum |a_n| < \infty$, show that the series would define a continuous periodic function on $\mathbf{R}$.)