A Correction Note: Attractive Nearest Neighbor Spin Systems on the Integers

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Abstract

In this note, we discuss a proof in T. Liggett's work on attractive translation invariant nearest neighbor spin systems on the integers. A correction to a wrong estimate is provided.

1   Introduction

For the purposes of this note, an attractive translation invariant nearest neighbor spin system is a certain kind of Feller process defined on the compact state space $X = \{0,1\}^{\mathbb{Z}}$ with rates satisfying the attractiveness inequalities, and depending only on the nearest neighbors. (See [2], p.144-145 for more details.)

T. Liggett proves in [1] and [2], (p. 152 Theorem 3.13) the following theorem:

Theorem 1. *All attractive translation invariant nearest neighbor spin systems on the integers have only the minimal and maximal invariant measures (ordered stochastically) as the extremal invariant measures.*

We discuss the estimate in [2] of lemma 3.7 part (e). It is wrong, but the similar estimate $\epsilon \int g_{m,n}^{l+1} d\nu \leq (4Kl + 2\epsilon) \int g_{m,n}^{l} d\nu$ is valid. This change does not affect the proof moving forward, because the only time the estimate is used is in the proof of lemma 3.10 in [2], where it is used to derive that $\sup_{m \leq n} \int g_{m,n}^{l} d\nu < \infty$. This fact still holds with the new estimate. The same line of reasoning appears in [1] within the proof of lemma 2.2 there.

We will refer to [2] from now on.

2   A Correction

The problem with the estimate as written is at the top of p.150 of [2] in the discussion of bounding below the positive contribution to $\tilde{\Omega}g_{m,n}^{l}$. The argument there fails to consider the possibility that change in the γ coordinate at any of the x_i among the left and right endpoints in the $l+1$ length intervals may not only create a length l interval, but also destroy an adjacent length l interval. But this can only happen at at most $2g_{m,n}^{l}$ such sites, i.e. the left and right

neighbors of intervals of length l. The bound below on the rate of the type of flip in question is still correctly stated as ϵ so as long as we replace $\epsilon(g_{m,n}^{l+1})$ by $\epsilon(g_{m,n}^{l+1} - 2g_{m,n}^l)$ the estimate is correct. This results in the display reading

$$\tilde{\Omega}g_{m,n}^l \geq \epsilon(g_{m,n}^{l+1} - 2g_{m,n}^l) - 4Klg_{m,n}^l.$$

References

[1] Liggett, T. (1978). Attractive Nearest Neighbor Spin Systems on the Integers. Ann. Probability Vol. 6 No. 4, p.629-636.

[2] Liggett, T. (1985). Interacting Particle Systems. Springer Berlin Heidelberg 2005. p.143-152 and Ch. 1.