T. Liggett Mathematics 31A – First Midterm – Solutions January 30, 2009

(15) 1. Evaluate the following limits:

$$\lim_{h\to 0}\frac{(1+h)^3-1}{h}=\lim_{h\to 0}\frac{3h+3h^2+h^3}{h}=\lim_{h\to 0}(3+3h+h^2)=3.$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}=\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}\Big(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\Big)$$

$$=\lim_{x\to 0}\frac{2x}{x(\sqrt{1+x}+\sqrt{1-x})}=1.$$

(10) 2. Determine the point(s) at which the function

$$f(u)=\frac{u+1}{u^2-9}$$

is discontinuous. For each such point a, find $\lim_{u\to a+} f(u)$.

Solution: The function is continuous at every point, except possibly points at which the denominator is 0, which are $a=\pm 3$. Near each of these points, the denominator is close to 0, but the numerator is not. Therefore, the limits are $\pm\infty$. To decide the correct sign, look at the signs of the numerator and denominator for u close to a:

$$\lim_{u\to -3+} f(u)=+\infty,\quad \lim_{u\to 3+} f(u)=+\infty.$$

(14) 3. Compute the following derivatives:

$$(a)\ \frac{d}{dx}(x^2\cos x)=2x\cos x-x^2\sin x$$

$$(b)\ \frac{d}{dt}\Big(\frac{3t^2+1}{5t^3+7t}\Big)=\frac{(5t^3+7t)(6t)-(3t^2+1)(15t^2+7)}{(5t^3+7t)^2}=\frac{-15t^4+6t^2-7}{t^2(5t^2+7)^2}$$

(11) 4. Let $f(x)=3x^2+1$ and $g(x)=2x^3-12x$. Find the values of x for which the tangent line to the graph of f at $(x,f(x))$ and the tangent line to the graph of g at $(x,g(x))$ are parallel.

Solution: Need to find those x's for which $f'(x)=g'(x)$, since this means the two tangent lines have the same slope. So, need to solve $6x=6x^2-12$ for x. The solutions are -1 and 2.

(11) 5. The position of a car at time t is given by

$$s(t)=t^3-6t^2+15t+1.$$

Find all times t at which the car is accelerating.

Solution: The acceleration at time t is $s''(t) = 6t - 12$. This is positive if and only if $t > 2$.

(14) 6. Differentiate the following functions:

$$\frac{d}{dx}\sqrt{4 - 3\cos x} = \frac{1}{2}(4 - 3\cos x)^{-1/2}\frac{d}{dx}(4 - 3\cos x) = \frac{3\sin x}{2\sqrt{4 - 3\cos x}}.$$

$$\frac{d}{dx}\tan(x^2) = 2x\sec^2(x^2).$$

(10) 7. Find the equation for the tangent line to the curve

$$x^2y^3 + 2y = 3x$$

at the point $(2, 1)$.

Solution: Differentiate implicitly with respect to x to get

$$2xy^3 + 3x^2y^2y' + 2y' = 3.$$

When $x = 2, y = 1$, this gives $y' = -1/14$. So, the equation for the tangent line is $y - 1 = -(1/14)(x - 2)$, or

$$y = -\frac{1}{14}x + \frac{8}{7}.$$

(15) 8. Prove the following:

$$\lim_{h \to 0} \frac{\sin h}{h} = 1.$$

See page 80 of the text.