Mathematics 275C – HW1 – Due Wednesday, April 13, 2011.

Problems from the text: Chapter 1 # 1.31 (students who did this problem last Spring need not do it again); Chapter 2 # 2.5, 2.7.

In the following problems, the chains are discrete time.

A. Consider the simple random walk on Z^1 with transition probabilities $p(x, x+1) = p, p(x, x-1) = q$, where $p + q = 1, 0 < p < 1$.
(a) Find all stationary measures for this chain.
(b) Explain why this implies that the chain is transient if $p \neq \frac{1}{2}$.

B. Give an example of a Markov chain that has no nonzero stationary measure.

C. Suppose X_n is a null recurrent Markov chain. Show that

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} p_k(x, y) = 0, \quad x, y \in S, \tag{1}$$

using the following outline:
(a) Fix $x \in S$ and define

$$\pi_n(y) = \frac{1}{n} \sum_{k=1}^{n} p_k(x, y).$$

Explain why there is a subsequence n_i so that $\pi(y) = \lim_{i\to\infty} \pi_{n_i}(y)$ exists for each y.
(b) What can you say about $\sum_y \pi(y)$ and $\pi(z) - \sum_y \pi(y)p(y, z)$?
(c) Conclude that $\pi \equiv 0$, and that (1) holds.