Mathematics 275B – HW5 – Due Friday March 13, 2009.

This assignment is to be done WITHOUT consultation. You may use your notes and Durrett's book, but may not discuss these problems with anyone or use other sources.

A. Problems from the text: Chapter 4 # 5.5, 6.4; Chapter 6 # 1.3, 1.6, 7.1, 7.2 (these are based on material to be covered in class on Wednesday, March 11)

B1. (This problem outlines the proof of the convergence theorem without using upcrossings. Needless to say, you should not use any results that are based on the upcrossing lemma.)

(a) Suppose that X_n is a square integrable martingale. Show that $E(X_n - X_m)^2 = EX_n^2 - EX_m^2$ for $m < n$.

(b) Suppose that X_n is a martingale satisfying $\sup_n EX_n^2 < \infty$. Show that X_n converges to some X in L_2.

(c) Use Doob's inequality to show that the convergence in (b) occurs a.s. also.

B2. Suppose that $S_n = X_1 + \cdots + X_n$, where X_i are i.i.d., and $t > 0$ satisfies $Ee^{tX_1} = 1$. Show that

$$P(S_n \geq x \text{ for some } n \geq 1) \leq e^{-tx}, \quad x > 0.$$

B3. Prove the following: If $\{X_n, n \geq 1\}$ is uniformly integrable, then so is $\{Y_n, n \geq 1\}$, where $Y_n = \frac{1}{n}\sum_{k=1}^{n} X_k$.