Mathematics 171 – HW9 – Due Thursday, June 2, 2011.

Problems 8.20,8.21,8.25,8.27,8.28,8.30,8.32,8.35,8.37 on pages 203-206 plus the following:

Q. Let $p_t(x, y)$ be the transition probabilities for an irreducible continuous time Markov chain on a finite set $S = \{1, \ldots, N\}$ A collection of indistinguishable particles moving independently on S can be thought of as a Markov chain W_t on the set of configurations of particles on S. So, for example, if $N = 4$, then $(5, 7, 1, 0)$ corresponds to the configuration in which there are 5 particles at site 1, 7 particles at site 2, 1 particle at site 3, and none at site 4.

(a) Suppose W_0 has the distribution in which $\{W_0(i), 1 \leq i \leq N\}$ are independent, with $W_0(i)$ having the Poisson distribution with parameter $\lambda(i)$. Use the properties of thinning and superposition of Poisson random variables to determine the distribution of W_t.

(b) Using (a), find some stationary distributions for W_t.

(c) The chain W_t is not irreducible, since the total number of particles does not change. Find the closed irreducible classes for W_t.

(d) For each irreducible class, find the stationary distribution for the chain restricted to that class.