

Mathematics 171 – HW6 – Due Thursday, May 12, 2011.

Problems 5.7 and 5.13, on pages 121-122, problems 7.4 and 7.8 on page 152, plus the following:

K. (a) Let X_n be a nonnegative martingale. Use the stopping time theorem to show that for $\lambda > 0$,

$$P\left(\max_{0 \leq m \leq n} X_m > \lambda\right) \leq \frac{EX_n}{\lambda}.$$

(b) Explain why this is an improvement over Markov's inequality.

L. In the context of problem 5.7, find the limiting distribution of X_n as $n \rightarrow \infty$. (This is a special case of the result that I mentioned without proof at the end of class on 5/4.)

M. (a) In the context of Problem 5.16, show that $\phi(X_n)$ is a martingale.
(b) Do Problem 5.16.

N. Suppose $M_n = X_1 + \cdots + X_n$ (with $M_0 = 0$) is a martingale.

(a) Show that $EX_k = 0$ for each k .

(b) Show that $Var(M_n) = Var(X_1) + \cdots + Var(X_n)$.