Mathematics 170A – HW9 – Due Tuesday, March 13, 2012.

Problems 11,15,16 on pages 188-190.

K_1. Let X be uniform on $[0,1]$ and $Y = 4X(1-X)$. Find the CDF and PDF of Y.

K_2. Let X be uniform on $[0,1]$ and $Y = -\log X$. What is the distribution of Y?

K_3. The weight of a person chosen at random from a population is normally distributed with mean μ and variance σ^2. Suppose that $P(X \leq 160) = \frac{1}{2}$ and $P(X \leq 140) = \frac{1}{4}$.
(a) Find μ and σ.
(b) Find $P(X \geq 200)$.
(c) Among all people in the population weighing at least 200 pounds, what percentage weigh over 200 pounds?

K_4. Let X and Y be continuous random variables with joint distribution function $F(x,y)$ and joint density function $f(x,y)$. Find the joint distribution function $G(x,y)$ and joint density function $g(x,y)$ of the random variables $W = X^2$ and $Z = Y^2$.

K_5. Let X and Y be continuous random variables with joint density function

$$f(x,y) = \begin{cases} \lambda^2 e^{-\lambda y} & \text{if } 0 \leq x \leq y; \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the marginal densities of X and Y.
(b) Find the joint distribution function of X and Y.

K_6. Let R and Θ be independent random variables. Suppose that Θ is uniform on $(-\pi, \pi)$ and R has PDF

$$f(r) = \begin{cases} re^{-r^2/2} & \text{if } r > 0; \\ 0 & \text{otherwise.} \end{cases}$$

Let $X = R\cos\Theta$ and $Y = R\sin\Theta$.
(a) Find the joint PDF of (X,Y).
(b) What are the distributions of X and Y?
(c) Are X and Y independent? Explain.

K_7. Let $Y \mid \Lambda$ be exponentially distributed with parameter Λ, where Λ has the Gamma density with parameters $\alpha > 0$ and $\beta > 0$:

$$f(x) = \begin{cases} \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} & \text{for } x > 0; \\ 0 & \text{for } x \leq 0. \end{cases}$$

(Here

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx,$$

though you don't need to know that to do the problem.)
(a) Find the marginal density of Y.
(b) Find the conditional density of Λ given $Y = y$.