Mathematics 170A – HW7 – Due Tuesday, February 28, 2012.

Problems 38, 40, 41(a,b,c), 42(a,b) on pages 132-133 and problem 1 on page 184.

I_1. Let X_1 and X_2 be independent geometric random variables with parameters p_1 and p_2 respectively.
(a) Find $P(X_1 \geq X_2)$.
(b) Find $P(X_1 = X_2)$.

I_2. Let X_1 and X_2 be independent geometric random variables with parameters p_1 and p_2 respectively. Let $D = X_1 - X_2$ and $M = \min(X_1, X_2)$.
(a) Find the joint PMF of D and M.
(b) Find the marginal PMF's of D and M.
(c) Are D and M independent? Explain.

I_3. Let X_1 and X_2 be independent Poisson random variables with parameters λ_1 and λ_2 respectively. What is the distribution of $S = X_1 + X_2$?

I_4. Let X_1 and X_2 be independent random variables that are uniformly distributed on $\{1, ..., n\}$. What is the PMF of $S = X_1 + X_2$?

I_5. The random variables $X_1, ..., X_n$ are independent, and X_i has mean μ (independent of i) and variance σ_i^2 (depending on i). In a common statistical setting, μ is regarded as unknown, while the σ_i's are known. It is proposed to estimate μ by the value of the random variable $S = \sum_i a_i X_i$ for some choice of constants a_i.
(a) Under what condition on the a_i's is $ES = \mu$?
(b) Among all choices of the a_i's that satisfy $ES = \mu$, find the one that minimizes $\text{var}(S)$.