

Mathematics 170A – HW6 – Due Tuesday, February 21, 2012.

Problems 24,25,26,31(use conditioning, not problem 27, which I will discuss in class 2/15),32 on pages 124-133, and problem 17 on page 249.

H_1 . Suppose n balls are distributed at random into r boxes. Let

$$X_i = \begin{cases} 1 & \text{if box } i \text{ is empty;} \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Compute EX_i .
- (b) For $i \neq j$, compute EX_iX_j and $\text{Cov}(X_i, X_j)$.
- (c) Let $S =$ the number of empty boxes. Relate S to the X_i 's, and use part (a) to compute ES and $\text{Var}(S)$.

H_2 . Toss a biased coin (with probability p of heads and $1 - p$ of tails) until the first head appears. Let N be the number of tosses required to get the first head. Now toss a fair die N times, and let S be the sum of results of the N tosses of the die. Use conditioning to compute ES .

H_3 . A miner is trapped in a mine containing three doors. The first door leads to a tunnel that leads him to safety after three hours. The second door leads him back to the mine after five hours. The third door leads him back to the mine after seven hours. Each time, the miner is equally likely to choose each of the three doors. Let X be the number of hours it takes him to get to safety, and Y the number of the door he initially chooses.

- (a) Compute $E[X|Y = i]$ for $i = 1, 2, 3$ in terms of EX .
- (b) Compute EX .

H_4 . Consider an experiment that has three possible outcomes that occur with probabilities p_1, p_2, p_3 . Suppose that n independent repetitions of the experiment are made, and let X_i denote the number of times that outcome i occurs ($i = 1, 2, 3$).

- (a) What is the distribution of $X_1 + X_2$?
- (b) Compute $P(X_2 = k | X_1 + X_2 = m)$.
- (c) What is $E(X_2 | X_1 + X_2 = m)$?