Mathematics 170A – HW2 – Due Tuesday, January 24, 2012.

Problems 14,16,17,18,19,21,22,23 on pages 56-58.

C. A community has m families with children. The largest family has k children. For $i = 1, ..., k$, there are n_i families with i children, so $n_1 + n_2 + \cdots + n_k = m$. Choose a child at random from the community in one of the following ways:

1. Choose one of the m families at random, and then choose one of the children at random from that family.

2. Choose one of the $n_1 + 2n_2 + \cdots + kn_k$ children at random.

(a) What is the probability that the child chosen is the first-born in his/her family if you use method 1?

(b) What is the probability that the child chosen is the first-born in his/her family if you use method 2?

(c) Which method results in the larger probability that the child chosen is the first-born in his/her family?

D. A coin has probability p of coming up heads and $1 - p$ of coming up tails. Let x_n be the probability that if the coin is tossed n times, the number of heads obtained is even.

(a) Show that $x_n = p(1 - x_{n-1}) + (1 - p)x_{n-1}$ for $n \geq 1$.

(b) Let $a_n = 2x_n - 1$. Express a_n in terms of a_{n-1}.

(c) Use part (b) to compute a_n, and therefore x_n, for $n \geq 1$.