T. Liggett Mathematics 131AH – Midterm November 4, 2009

1	2	3	4	5	6	7	Total

Last name: First name:

(15) 1. (a) Define: "x is a limit point of E".

Let E' be the set of limit points of E. Decide whether each of the following statements is true or false. If true, prove it; if false, give a counterexample.

(b) $(E \cap F)' \subset E' \cap F'$

(c) $(E \cap F)' \supset E' \cap F'$

(15) 2. Let $\mathcal{N} = \{1, 2, ...\}$ be the set of natural numbers, $\mathcal{F}$ be the collection of finite subsets of $\mathcal{N}$, and $\mathcal{S}$ be the collection of all subsets of $\mathcal{N}$.

(a) Prove that $\mathcal{F}$ is countable.

(b) Find a one-to-one correspondence between $\{0, 1\}^{\mathcal{N}}$ and $\mathcal{S}$.

(c) Is $\mathcal{S}$ countable or uncountable? Why?

(15) 3. Decide whether each of the following statements is true or false. No explanation is needed.

(a) Q, the set of rationals, has the least upper bound property. Answer:

(b) The Cantor set contains both rational and irrational points. Answer:

(c) Every metric space has at most finitely many subsets that are both open and closed. Answer:

(d) Every subset of a general metric space that is closed and bounded is compact. Answer:

(e) The union of two compact sets is compact. Answer:

(15) 4. (a) Define "K is compact".

(b) Prove that every compact set is bounded.

(c) Prove directly from the definition that $[0, 1)$ is not compact in R^1.

(20) 5. For each part, give an example of a set A in R^1 with the usual metric that has the required properties. There is no need to prove that it has those properties:

(a) A is countable and compact.

(b) A and A^c are both dense.

(c) A is not connected, but $\overline{A}$ is.

(d) A is countable and has no limit points.

(10) 6. Suppose $A \subset R^1$ is uncountable. Prove that A has a limit point.

(10) 7. Show that for each x, $\mathcal{O} = \{y \in X : d(x, y) > 1\}$ is open.