MATH 31B HW#5

§8.8

5. $\int_1^\infty \frac{1}{(3x+1)^2}dx = \lim_{t\to\infty}\int_1^t \frac{1}{(3x+1)^2}dx = \lim_{t\to\infty} -\frac{1}{3}\cdot\frac{1}{3x+1}\Big]_1^t = -\frac{1}{3}\lim_{t\to\infty}\left(\frac{1}{3t+1}-\frac{1}{4}\right) = \boxed{\frac{1}{12}}$

6. $\int_{-\infty}^0 \frac{1}{(2x-5)}dx = \lim_{t\to-\infty}\int_0^t \frac{1}{2x-5}dx = \lim_{t\to-\infty}\frac{1}{2}\ln|2x-5|\Big]_t^0$

$= \frac{1}{2}\ln 5 - \lim_{t\to-\infty}\frac{1}{2}\ln|2t-5| = \frac{1}{2}\ln 5 - \infty = \boxed{-\infty}$ divergent

33. $\int_0^{33}(x-1)^{-1/5}dx = \int_0^1 (x-1)^{-1/5}dx + \int_1^{33}(x-1)^{-1/5}dx$

$\int_0^1 (x-1)^{-1/5}dx = \lim_{t\to1^-}\int_0^t (x-1)^{-1/5}dx = \lim_{t\to1^-}\frac{5}{4}(x-1)^{4/5}\Big]_0^t = -\frac{5}{4}$

$\int_1^{33}(x-1)^{-1/5}dx = \lim_{t\to1^+}\int_t^{33}(x-1)^{-1/5}dx = \lim_{t\to1^+}\frac{5}{4}(x-1)^{4/5}\Big]_t^{33} = 20$

$\Rightarrow \int_0^{33}(x-1)^{-1/5}dx = \boxed{\frac{75}{4}}$

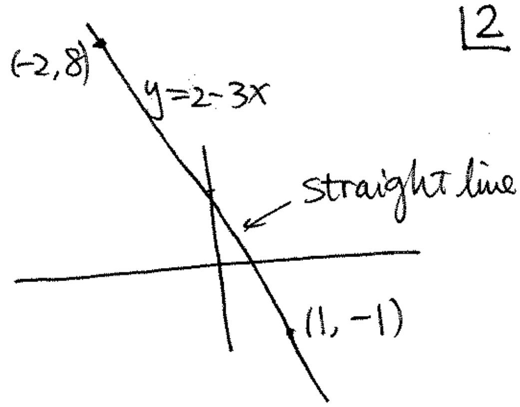
34. $\int_0^1 \frac{1}{4y-1}dy = \int_0^{1/4}\frac{1}{4y-1}dy + \int_{1/4}^1 \frac{1}{4y-1}dy$

$\int_0^{1/4}\frac{1}{4y-1}dy = \lim_{t\to 1/4^-}\int_0^t \frac{1}{4y-1}dy = \lim_{t\to1/4^-}\frac{1}{4}\ln|4y-1|\Big]_0^t$

$= \lim_{t\to1/4^-}\frac{1}{4}\ln|4t-1| = -\infty$

$\Rightarrow \int_0^1 \frac{1}{4y-1}dy$ is divergent.

§9.1



1. $L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

$y = 2-3x \quad , \quad \frac{dy}{dx} = -3$

$\Rightarrow L = \int_{-2}^{1} \sqrt{1+3^2}\,dx = \sqrt{10}\,(1-(-2)) = \boxed{3\sqrt{10}}$

$d = \sqrt{(-2-1)^2+(8-(-1))^2} = \sqrt{90} = \boxed{3\sqrt{10}} = L$

5. $y = 1+6x^{3/2} \quad , \quad 0 \le x \le 1$

$\frac{dy}{dx} = 9x^{1/2} \quad ; \quad L = \int_0^1 \sqrt{1+(9x^{1/2})^2}\,dx = \int_0^1 \sqrt{1+81x}\,dx$

$= \frac{2}{3}(81x+1)^{3/2}\cdot\frac{1}{81}\Big]_0^1 = \boxed{\frac{2}{243}\left(82\sqrt{82}-1\right)}$

6. $y^2 = 4(x+4)^3 \quad 0 \le x \le 2, \quad y>0$

$y = 2(x+4)^{3/2} \quad (y>0)$

$\frac{dy}{dx} = 3(x+4)^{1/2}$

$L = \int_0^2 \sqrt{1+9(x+4)}\,dx = \int_0^2 (9x+37)^{1/2}dx = \frac{1}{9}\cdot\frac{2}{3}(9x+37)^{3/2}\Big]_0^2$

$= \boxed{\frac{2}{27}\left(55\sqrt{55}-37\sqrt{37}\right)}$

7. $y = \frac{x^5}{6} + \frac{1}{10x^3} \qquad 1 \le x \le 2$

$\frac{dy}{dx} = \frac{5}{6}x^4 - (3/10)x^{-4}$

$L = \int_1^2 \sqrt{1+\left(\frac{5}{6}x^4 - \frac{3}{10x^4}\right)^2}\,dx = \int_1^2 \sqrt{1+\left(\frac{5}{6}x^4\right)^2+\left(\frac{3}{10x^4}\right)^2-\frac{1}{2}}\,dx$

$= \int_1^2 \sqrt{\left(\frac{5x^4}{6}\right)^2+\left(\frac{3}{10x^4}\right)^2+\frac{1}{2}}\,dx = \int_1^2 \sqrt{\left(\frac{5}{6}x^4+\frac{3}{10x^4}\right)^2}\,dx = \int_1^2 \left|\frac{5x^4}{6}+\frac{3}{10x^4}\right|dx$

$= \int_1^2 \left(\frac{5x^4}{6}+\frac{3}{10x^4}\right)dx = \frac{x^5}{6} - \frac{1}{10x^3}\Big]_1^2 = \left(\frac{32}{6}-\frac{1}{80}\right)-\left(\frac{1}{6}-\frac{1}{10}\right) = \boxed{\frac{1261}{240}}$

8. $y = \frac{x^2}{2} - \frac{\ln x}{4}$, $2 \le x \le 4$

$\frac{dy}{dx} = x - \frac{1}{4x}$

$L = \int_2^4 \sqrt{1 + (x - \frac{1}{4x})^2}\, dx = \int_2^4 \sqrt{1 + x^2 + (\frac{1}{4x})^2 - \frac{1}{2}}\, dx = \int_2^4 \sqrt{(x + \frac{1}{4x})^2}\, dx$

$= \int_2^4 |x + \frac{1}{4x}|\, dx = \int_2^4 (x + \frac{1}{4x})\, dx$ (since $x + \frac{1}{4x} > 0$ for $2 \le x \le 4$)

$= \frac{x^2}{2} + \frac{\ln|x|}{4}\Big]_2^4 = \frac{x^2}{2} + \frac{\ln x}{4}\Big]_2^4 = \boxed{6 + \frac{\ln 2}{4}}$

9. $x = \frac{1}{3}\sqrt{y}(y-3)$, $1 \le y \le 9$

$\frac{dx}{dy} = \frac{1}{2}y^{\frac{1}{2}} - \frac{1}{2}y^{-\frac{1}{2}} = \frac{1}{2}(\sqrt{y} - \frac{1}{\sqrt{y}})$

$L = \int_1^9 \sqrt{1 + \frac{1}{4}(\sqrt{y} - \frac{1}{\sqrt{y}})^2}\, dy = \int_1^9 \sqrt{\frac{y}{4} + \frac{1}{4y} + 1 - \frac{1}{2}}\, dy = \int_1^9 \sqrt{(\frac{\sqrt{y}}{2} + \frac{1}{2\sqrt{y}})^2}\, dy$

$= \int_1^9 |\frac{1}{2}(\sqrt{y} + \frac{1}{\sqrt{y}})|\, dy = \frac{1}{2}\int_1^9 (\sqrt{y} + \frac{1}{\sqrt{y}})\, dy = \frac{1}{3}y^{\frac{3}{2}} + y^{\frac{1}{2}}\Big]_1^9 = \boxed{\frac{32}{3}}$

10. $y = \ln(\cos x)$ $\quad 0 \le x \le \pi/3$

$\frac{dy}{dx} = \frac{1}{\cos x}(-\sin x) = -\tan x$

$L = \int_0^{\pi/3} \sqrt{1 + \tan^2 x}\, dx = \int_0^{\pi/3} \sqrt{\sec^2 x}\, dx = \int_0^{\pi/3} |\sec x|\, dx = \int_0^{\pi/3} \sec x\, dx$

$= \ln|\sec x + \tan x|\Big]_0^{\pi/3} = \boxed{\ln(2 + \sqrt{3})}$

11. $y = \ln(\sec x)$ $\quad 0 \le x \le \pi/4$

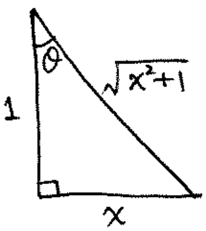
$\frac{dy}{dx} = \frac{1}{\sec x}\sec x \tan x = \tan x$ $\quad$ [NOTE here: $\ln(\sec x) = \ln(\frac{1}{\cos x}) = -\ln(\cos x)$

$L = \int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx = \ln|\sec x + \tan x|\Big]_0^{\pi/4} = \boxed{\ln(\sqrt{2} + 1)}$

The functions in #10 & #11 are symmetric about x-axis.

12. $y = \ln x \qquad 1 \le x \le \sqrt{3}$

$$\frac{dy}{dx} = \frac{1}{x}$$



$\tan\theta = x$

$\Rightarrow dx = \sec^2\theta\, d\theta$

$\cos\theta = \frac{1}{\sqrt{x^2+1}}$

$\theta = \tan^{-1} x$

$x: 1 \to \sqrt{3} \Leftrightarrow \theta: \frac{\pi}{4} \to \frac{\pi}{3}$

$$L = \int_1^{\sqrt{3}} \sqrt{1+\frac{1}{x^2}}\, dx = \int_1^{\sqrt{3}} \frac{\sqrt{x^2+1}}{x}\, dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\cos\theta} \cdot \frac{1}{\tan\theta} \cdot \sec^2\theta\, d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\cos^2\theta \sin\theta}\, d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin\theta\, d\theta}{\cos^2\theta \sin^2\theta} = \int_{\pi/4}^{\pi/3} \frac{\sin\theta\, d\theta}{\cos^2\theta(1-\cos^2\theta)}$$

let $u = \cos\theta$, $du = -\sin\theta\, d\theta$

$\theta: \frac{\pi}{4} \to \frac{\pi}{3} \Leftrightarrow u: \frac{\sqrt{2}}{2} \to \frac{1}{2}$

$$= \int_{\sqrt{2}/2}^{1/2} \frac{-du}{u^2(1-u^2)} = \int_{\sqrt{2}/2}^{1/2} \frac{du}{u^2(u^2-1)}$$

write $\frac{1}{u^2(u^2-1)} = \frac{A}{u} + \frac{B}{u^2} + \frac{C}{u+1} + \frac{D}{u-1}$

$\Rightarrow 1 = Au(u^2-1) + B(u^2-1) + Cu^2(u-1) + Du^2(u+1)$

$u=0: \ B = -1$

$u=1: \ D = \frac{1}{2}$

$u=-1: \ C = -\frac{1}{2}$

$u=2: \ A = 0$

$$= \int_{\sqrt{2}/2}^{1/2} -u^{-2}\, du + \int_{\sqrt{2}/2}^{1/2} \left(-\frac{1}{2}\right)\frac{du}{u+1} + \int_{\sqrt{2}/2}^{1/2} \frac{1}{2}\,\frac{du}{u-1}$$

$$= u^{-1}\Big]_{\sqrt{2}/2}^{1/2} - \frac{1}{2}\ln|u+1|\Big]_{\sqrt{2}/2}^{1/2} + \frac{1}{2}\ln|u-1|\Big]_{\sqrt{2}/2}^{1/2}$$

$$= 2-\sqrt{2} - \frac{1}{2}\ln\frac{\frac{1}{2}+1}{\frac{\sqrt{2}}{2}+1} + \frac{1}{2}\ln\frac{1-\frac{1}{2}}{-\frac{\sqrt{2}}{2}+1}$$

$$= 2-\sqrt{2} - \frac{1}{2}\ln\frac{3}{\sqrt{2}+2} + \frac{1}{2}\ln\frac{1}{-\sqrt{2}+2}$$

$$= 2-\sqrt{2} - \frac{1}{2}\left(\ln\frac{3}{2+\sqrt{2}} - \ln\frac{1}{2-\sqrt{2}}\right)$$

$$= 2-\sqrt{2} - \frac{1}{2}\ln\frac{3(2-\sqrt{2})}{2+\sqrt{2}}$$

$$= 2-\sqrt{2} - \frac{1}{2}\ln\frac{3(2-\sqrt{2})^2}{2}$$

$$= 2-\sqrt{2} - \frac{1}{2}\ln 3 - \frac{1}{2}\ln\frac{(2-\sqrt{2})^2}{(\sqrt{2})^2}$$

$$\boxed{\begin{aligned} &= 2-\sqrt{2} - \tfrac{1}{2}\ln 3 - \ln(\sqrt{2}-1) \\ \text{or } &= 2-\sqrt{2} - \tfrac{1}{2}\ln 3 + \ln(\sqrt{2}+1) \end{aligned}}$$

since $\frac{1}{\sqrt{2}-1} = \sqrt{2}+1$

NOTE: Since $u = \cos\theta$, $\theta = \tan^{-1}x$, we have $u = \cos(\tan^{-1}x) = \frac{1}{\sqrt{x^2+1}}$

So, we've shown that $\int \sqrt{1+\frac{1}{x^2}}\, dx = \sqrt{x^2+1} - \frac{1}{2}\ln\left|\frac{1}{\sqrt{1+x^2}}+1\right| + \frac{1}{2}\ln\left|\frac{1}{\sqrt{x^2+1}}-1\right| + C$

$= \sqrt{x^2+1} - \frac{1}{2}\ln\left|\frac{1+\sqrt{x^2+1}}{\sqrt{x^2+1}-1}\right| + C = \sqrt{x^2+1} - \frac{1}{2}\ln\left|\frac{(1+\sqrt{x^2+1})^2}{x^2}\right| + C$

$= \sqrt{x^2+1} - \ln\left|\frac{1+\sqrt{x^2+1}}{x}\right| + C \qquad (*) \qquad$ (or, see the formula on the back cover =))

3. $y = \cosh x \qquad 0 \le x \le 1$

$\frac{dy}{dx} = \sinh x$

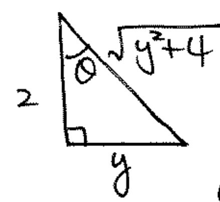
$L = \int_0^1 \sqrt{1+\sinh^2 x}\, dx = \int_0^1 \sqrt{1+\left(\frac{e^x}{2}-\frac{e^{-x}}{2}\right)^2}\, dx = \int_0^1 \sqrt{\left(\frac{e^x}{2}\right)^2+\left(\frac{e^{-x}}{2}\right)^2+1-\frac{1}{2}}\, dx$

$= \int_0^1 \sqrt{\left(\frac{e^x}{2}\right)^2+\frac{1}{2}+\left(\frac{e^{-x}}{2}\right)^2}\, dx = \int_0^1 \sqrt{\left(\frac{e^x}{2}+\frac{e^{-x}}{2}\right)^2}\, dx = \int_0^1 |\cosh x|\, dx = \int_0^1 \cosh x\, dx$

$= \sinh x \Big]_0^1 = \boxed{\sinh 1 \quad \text{or} \quad \frac{e-e^{-1}}{2}}$

14. $y^2 = 4x \qquad 0 \le y \le 2$

$x = \frac{y^2}{4} \Rightarrow \frac{dx}{dy} = \frac{y}{2}$



$\tan\theta = \frac{y}{2}$

$dy = 2\sec^2\theta\, d\theta$

$\cos\theta = \frac{2}{\sqrt{y^2+4}}$

$L = \int_0^2 \sqrt{1+\frac{y^2}{4}}\, dy = \int_0^2 \frac{1}{2}\sqrt{y^2+4}\, dy$

$y: 0 \to 2 \Leftrightarrow \theta: 0 \to \frac{\pi}{4}$

$= \int_0^{\pi/4} \frac{1}{2}\frac{2}{\cos\theta} 2\sec^2\theta\, d\theta = \int_0^{\pi/4} \frac{2}{\cos^3\theta}\, d\theta$

$= \int_0^{\pi/4} \frac{2\cos\theta}{\cos^2\theta\cdot\cos^2\theta}\, d\theta = \int_0^{\pi/4} \frac{2\cos\theta\, d\theta}{(1-\sin^2\theta)^2}$ let $u = \sin\theta$. $du = \cos\theta\, d\theta$

$\theta: 0 \to \frac{\pi}{4} \Leftrightarrow u: 0 \to \sqrt{2}/2$

$= \int_0^{\sqrt{2}/2} \frac{2\, du}{(1-u^2)^2}$

write $\frac{1}{(1-u^2)^2} = \frac{A}{1+u} + \frac{B}{(1+u)^2} + \frac{C}{1-u} + \frac{D}{(1-u)^2}$

$= \int_0^{\sqrt{2}/2} \frac{\frac{1}{2}du}{1+u} + \int_0^{\sqrt{2}/2} \frac{\frac{1}{2}du}{1-u}$

$\Rightarrow 1 = A(1+u)(1-u)^2 + B(1-u)^2 + C(1-u)(1+u)^2 + D(1+u)^2$

$u=-1: \quad B = 1/4 \qquad u = 1: \quad D = 1/4$

$+\int_0^{\sqrt{2}/2} \frac{1}{2}(1+u)^{-2}du + \int_0^{\sqrt{2}/2}\frac{1}{2}(1-u)^{-2}du$

$\left.\begin{array}{l} u=2: \quad A-3C = -\frac{1}{2} \\ u=3: \quad A-2C = -\frac{1}{4}\end{array}\right\} \Rightarrow A = C = 1/4$

$= \frac{1}{2}\ln|1+u|\Big]_0^{\sqrt{2}/2} - \frac{1}{2}\ln|1-u|\Big]_0^{\sqrt{2}/2}$

$-\frac{1}{2}\frac{1}{1+u}\Big]_0^{\sqrt{2}/2} + \frac{1}{2}\frac{1}{1-u}\Big]_0^{\sqrt{2}/2}$

$= \frac{1}{2}\ln\frac{2+\sqrt{2}}{2} - \frac{1}{2}\ln\frac{2-\sqrt{2}}{2} - \frac{1}{2}\left(\frac{2}{2+\sqrt{2}}-1\right) + \frac{1}{2}\left(\frac{2}{2-\sqrt{2}}-1\right)$

$= \frac{1}{2}\ln\frac{2+\sqrt{2}}{2}\frac{2}{2-\sqrt{2}} - \frac{1}{2}\frac{-\sqrt{2}}{2+\sqrt{2}} + \frac{1}{2}\frac{\sqrt{2}}{2-\sqrt{2}}$

$= \frac{1}{2}\ln\frac{2+\sqrt{2}}{2-\sqrt{2}} + \frac{1}{2}\frac{\sqrt{2}(2-\sqrt{2})}{2} + \frac{1}{2}\frac{\sqrt{2}(2+\sqrt{2})}{2}$

$= \frac{1}{2}\ln\frac{(2+\sqrt{2})^2}{2} + \frac{\sqrt{2}-1}{2} + \frac{\sqrt{2}+1}{2}$

$= \frac{1}{2}\ln\left(\frac{2+\sqrt{2}}{\sqrt{2}}\right)^2 + \sqrt{2} = \boxed{\ln(\sqrt{2}+1)+\sqrt{2}}$

You can also use the formula

$$\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln(u+\sqrt{a^2+u^2})$$

from the back cover to check your answer.

So, $L = \frac{1}{2}\int_0^2 \sqrt{y^2+4}\,dy = \frac{1}{2}\left[\frac{y}{2}\sqrt{y^2+4} + 2\ln(y+\sqrt{y^2+4})\right]_0^2$

$= \frac{1}{2}[2\sqrt{2} + 2\ln(2+2\sqrt{2}) - 2\ln 2]$

$= \boxed{\sqrt{2} + \ln(1+\sqrt{2})}$

5. $y = e^x \qquad 0 \le x \le 1$

Since we have already derived a formula in #12, we can use it here by solving

$\ln y = x \qquad e^0 \le y \le e^1 \qquad \frac{dx}{dy} = \frac{1}{y}$

so, $L = \int_0^1 \sqrt{1+e^{2x}}\,dx = \int_1^e \sqrt{1+\frac{1}{y^2}}\,dy$

$= \sqrt{y^2+1} - \ln\left|\frac{1+\sqrt{y^2+1}}{y}\right| \Bigg]_1^e$ by formula (✱) from #12

$= \sqrt{e^2+1} - \sqrt{2} - \ln\left|\frac{1+\sqrt{e^2+1}}{e}\right| + \ln(1+\sqrt{2})$

$= \boxed{\sqrt{e^2+1} - \sqrt{2} - \ln(1+\sqrt{e^2+1}) + 1 + \ln(1+\sqrt{2})}$

or $= \boxed{\sqrt{e^2+1} + 1 - \sqrt{2} - \ln(1+\sqrt{e^2+1}) - \ln(\sqrt{2}-1)}$ since $\frac{1}{\sqrt{2}-1} = \sqrt{2}+1$

§9.2

1. $y = \ln x \quad 1 \le x \le 3 \quad$ x-axis

$$\boxed{S = \int_1^3 2\pi \ln x \sqrt{1 + \frac{1}{x^2}}\, dx}$$

3. $y = \sec x \quad 0 \le x \le \pi/4 \quad$ y-axis

$$\boxed{S = \int_0^{\pi/4} 2\pi x \sqrt{1 + \sec^2 x \tan^2 x}\, dx}$$

OR. $y: 1 \to \sqrt{2} \quad x = \sec^{-1} y \quad \frac{dx}{dy} = \frac{1}{y\sqrt{y^2-1}}$

$$\boxed{S = \int_1^{\sqrt{2}} 2\pi \sec^{-1} y \sqrt{1 + \frac{1}{y^2(y^2-1)}}\, dy}$$

5. $y = x^3 \quad 0 \le x \le 2 \quad$ x-axis

$S = \int_0^2 2\pi x^3 \sqrt{1 + 9x^4}\, dx \qquad$ let $u = 1 + 9x^4$, $du = 36x^3 dx$

$x: 0 \to 2 \iff u: 1 \to 145$

$= \int_1^{145} 2\pi \cdot \frac{1}{36} u^{1/2} du$

$= \frac{\pi}{18} \cdot \frac{2}{3} \cdot u^{\frac{3}{2}} \Big]_1^{145} = \boxed{\frac{\pi}{27}\left(145\sqrt{145} - 1\right)}$

7. $y = \sqrt{x} \quad 4 \le x \le 9 \quad$ x-axis

$S = \int_4^9 2\pi \sqrt{x} \sqrt{1 + \left(\frac{1}{2\sqrt{x}}\right)^2}\, dx = \int_4^9 2\pi \sqrt{x + \frac{1}{4}}\, dx = 2\pi \frac{2}{3}\left(x + \frac{1}{4}\right)^{\frac{3}{2}} \Big]_4^9$

$= \frac{4\pi}{3}\left(\left(\frac{37}{4}\right)^{\frac{3}{2}} - \left(\frac{17}{4}\right)^{\frac{3}{2}}\right) = \frac{4\pi}{3}\left(\frac{37\sqrt{37}}{8} - \frac{17\sqrt{17}}{8}\right) = \boxed{\frac{\pi}{6}\left(37\sqrt{37} - 17\sqrt{17}\right)}$

9. $y = \cosh x, \quad 0 \le x \le 1, \quad$ x-axis

$S = \int_0^1 2\pi \cosh x \sqrt{1 + \sinh^2 x}\, dx = \int_0^1 2\pi \cosh^2 x\, dx \qquad$ (see #13 in §9.1)

$= \pi \int_0^1 [\cosh(2x) + 1]\, dx \qquad$ since $\cosh^2 x = \frac{\cosh(2x) + 1}{2}$

$= \pi \left[\frac{1}{2}\sinh(2x) + x\right]_0^1 \qquad$ b/c $\cosh(2x) = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1$

(see Pg 487) $\qquad \cosh^2 x - \sinh^2 x = 1$

$= \boxed{\pi\left[\frac{1}{4}(e^2 - e^{-2}) + 1\right]}$

1. $x = \frac{1}{3}(y^2+2)^{3/2}$. $1 \le y \le 2$. x-axis

$S = \int 2\pi y \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$ $\qquad \frac{dx}{dy} = \frac{1}{2}(y^2+2)^{\frac{1}{2}}(2y) = y(y^2+2)^{\frac{1}{2}}$

$= \int_1^2 2\pi y \sqrt{1+y^2(y^2+2)}\, dy = \int_1^2 2\pi y \sqrt{y^4+2y^2+1}\, dy = \int_1^2 2\pi y \sqrt{(y^2+1)^2}\, dy$

$= \int_1^2 2\pi y |y^2+1| dy = \int_1^2 \pi (2y)(y^2+1)\, dy = \int_{y=1}^{y=2} \pi (y^2+1)\, d(y^2+1)$

$= \pi \cdot \frac{(y^2+1)^2}{2}\Big]_1^2 = \boxed{\frac{21\pi}{2}}$

3. $y = \sqrt[3]{x}$, $1 \le y \le 2$. y-axis

$S = \int 2\pi x\, ds$ $\qquad x = y^3$. $\frac{dx}{dy} = 3y^2$

$= \int_1^2 2\pi y^3 \sqrt{1+9y^4}\, dy = \frac{\pi}{18}\int_1^2 36y^3 (1+9y^4)^{\frac{1}{2}} dy = \frac{\pi}{18} \cdot \frac{2}{3}(1+9y^4)^{3/2}\Big]_1^2$

$= \boxed{\frac{\pi}{27}(145\sqrt{145} - 10\sqrt{10})}$

5. $x = \sqrt{a^2 - y^2}$ $\qquad 0 \le y \le a/2$ $\qquad y$-axis

$\frac{dx}{dy} = \frac{1}{2}(a^2-y^2)^{-1/2}(-2y) = -y(a^2-y^2)^{-1/2}$

$S = \int_0^{a/2} 2\pi \sqrt{a^2-y^2} \cdot \sqrt{1+y^2(a^2-y^2)^{-1}}\, dy = \int_0^{a/2} 2\pi \sqrt{a^2-y^2+y^2}\, dy$

$= \int_0^{a/2} 2\pi |a|\, dy = \int_0^{a/2} 2\pi a\, dy$ $\qquad$ (since $a \ge 0$)

$= \boxed{\pi a^2}$

. $L_2 = 2\cdot(0.5+2.5) = 6$. $R_2 = 2(2.5+3.5) = 12$. $M_2 = 2(1.5+3.3) = 9.6$

underestimate overestimate overestimate (hard to tell)

$T_2 = f(0) + 2f(2) + f(4) = 0.5 + 5 + 3.5 = 9$, $T_2 < I$

$L_n < T_n < I < M_n < R_n$

2. For the given $f(x)$, and fixed n, $L_n > T_n$ [redacted] $> M_n > R_n$

(For simplicity, you can consider the case $n=2$)

Therefore, $R_n = 0.7811$, $M_n = 0.8632$, $T_n = 0.8675$ $L_n = 0.9540$

$0.8632 \le \int_0^2 f(x)\,dx \le 0.8675$

9. $\int_0^2 e^{-x^2}dx$. $n=10$. $\Delta x = 0.2$. let $f(x) = e^{-x^2}$

$T_{10} = 0.1\big(f(0) + 2f(.2) + 2f(.4) + \cdots + 2f(1.8) + f(2)\big) \approx 0.8818$

$M_{10} = 0.2\big(f(.1) + f(.3) + f(.5) + \cdots + f(1.9)\big) \approx 0.8822$

$f'(x) = e^{-x^2}(-2x)$. $f''(x) = e^{-x^2}(4x^2-2)$

since $|f''(x)| \le 2$ for $0 \le x \le 2$

$|E_T| \le \frac{2\cdot(2-0)^3}{12\cdot(10)^2} \approx 0.0133$ $|E_M| \le \frac{1}{2}|E_T| = 0.0067$

Need n so that $\frac{16}{12n^2} < 0.00001$ for Trapezoidal & $\frac{16}{24n^2} < 0.00001$

for midpt. $\Rightarrow n = 366$ for T_n & $n = 259$ for M_n

20. $\int_0^1 \cos(x^2)dx$ $n=8$ $\Delta x = 0.125$ let $f(x) = \cos(x^2)$

$T_8 = \frac{0.125}{2}\big(f(0) + 2f(.125) + 2f(.25) + \cdots + 2f(.875) + f(1)\big) \cong 0.9023$

$M_8 = 0.125\big(f(.0625) + f(.1875) + \cdots + f(.9375)\big) \cong 0.9056$

$f'(x) = -2x\sin(x^2)$ $f''(x) = -4x^2\cos x^2 - 2\sin x^2$

$|f''(x)| \le 2$ for $0 \le x \le 1$

$|E_T| \le \frac{2}{12\cdot 8^2} \approx 0.0026$ $|E_M| \le 0.0013$

T_n

$$\frac{2}{12n^2} < 0.00001$$

$n = 130$

M_n

$$\frac{2}{24n^2} < 0.00001$$

$n = 92$