# Public Goods, Uncertainty and Contingent Claims 

Esther Bruegger and Roberto H. Schonmann *<br>January 27, 2007


#### Abstract

A population of agents decides by majority rule on the realization of a discrete public good whose costs are common knowledge while its benefit is uncertain but is the same for all agents. Agents differ in their valuation of the public good, because they may have different levels of risk aversion or may differ in their expectations of the public good's benefit. Consequently, a majority might reject a beneficial public good if its costs are shared equally. We present a financing procedure which is budget balanced, easy to implement and to understand, and is individually rational, i.e., makes ex ante all agents better off: If the public good is financed through the dispostion of claims contingent on its realized benefit, all agents will then vote for the implementation of the public good. If all agents agree on the public good's expected benefit, only ex ante socially beneficial public goods are implemented.


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*Address: UCLA Mathematics Department, Box 951555, Los Angeles, CA 90095-1555. Email addresses: bruegger@math.ucla.edu, rhs@math.ucla.edu. The authors thank the NSF for financial support (grant number DMS 0300672).

## 1 Introduction

We consider a population of agents which decides by majority rule on the realization of public goods. We focus on the implementation of a discrete public good, that is, a public good which has a threshold production function: If its costs are not fully paid, it cannot be provided at all. The costs for the realization of the public good are common knowledge and once it is provided, each agent derives the same benefit from it. In this paper, we study situations in which this benefit is uncertain before the public good is realized. We assume that agents differ in their levels of risk aversion or in their expectations of what the public good's benefit will be. We suppose that information about the agents' risk preferences, respectively about their expectations, is private.

This paper addresses the case of a public good rejected by the majority in case costs are shared equally among the agents. We present a financing procedure for such a public good which is budget balanced, easy to implement and to understand, and is individually rational, i.e., makes ex ante all agents better off: If the public good is financed through the disposition of claims contingent on its realized benefit, all agents will then vote for the implementation of the public good. If all agents agree on the public good's expected benefit, only ex ante socially beneficial public goods are implemented.

To see how the claim works, let us look at an example: Shop owners in a mall discuss if they should enlarge an existing parking structure in order to attract more customers. Even if only a minority expects the gains from additional sales to be large enough to compensate them for their share of the costs and the risks, the financing through contingent claims could make the extension of the parking structure happen. The supporters are offered claims contingent on the additional sales made by all shops. The price they pay for the claims is used to pay for the construction costs. After the project is completed, the shop owners pay a fraction of their additional revenue to the claim buyers. Consequently, the claim buyers are taking the risk from the other shop owners and do so willingly because they expect to profit from the claims they hold as well as from the undertaken project.

Our setup is general and can be applied to many situations as long as the benefit to each agent can be measured or is the same for all agents. In many of
the most interesting applications of our model, the evaluation of the public good's benefits might be challenging. For instance, it would be a challenge to evaluate the benefits of a national AIDS prevention campaign. There will be savings in health related spendings as well as increased economic growth prospects because of the reduced number of disease related deaths. Assuming that these kinds of measurement challenges can be overcome, further examples for the claim solution include: A committee's decision whether to finance a research project, a society's decision whether to make a new vaccination mandatory, a community's decision whether to organize a nightly patrol, or the decision of a homeowners association whether to insulate their building in order to bring electricity costs down.

It is interesting to compare the contingent claims proposed in this paper with bonds issued by an authority, e.g., treasury bonds. Bonds do not address differences in level of risk aversion or differences in beliefs about the benefit from public goods. However, they do allow for the authority to concentrate the financing of a public good which has a delayed benefit on the hands of the agents who are more patient. Analogously, our contingent claims allow the authority to shift the financing towards those agents who are less risk averse or more optimistic about the outcome of the public good.

Our paper relates to the literature about voluntary provision of discrete public goods with incomplete information. Menezes et al. (2001) analyze the equilibria of the contribution game (contributions not refunded if project not completed) and subscription game (contributions refunded if project not completed) for discrete public goods: The probability that the agents in our model provide the public good voluntarily is strictly smaller than one in both the contribution and the subscription game. Moreover it is driven to zero if the number of agents becomes large (Mailath and Postlewaite, 1990). These results make a case for our claim solution since voluntary contribution seems to be a natural outside option for the agents if a project is rejected by the majority of agents.

## 2 The model

### 2.1 The setup

### 2.1.1 The political economy with a public good

We consider a population with $N$ agents. We assume that the population decides by majority vote on projects which have public good character. A project $\pi$ is realized at a total cost $C$ and yields a benefit $y$ to every agent. The nature of the benefit is such that no agent can be excluded from it. There exists an authority which implements the agents' decisions at no additional costs and does not have an own agenda. If a majority of agents votes for the implementation of a project, each agent bears an equal share of the costs, $c \equiv \frac{C}{N}$, and we assume these costs to be common knowledge. ${ }^{1}$

Agent $i$ has an initial endowment of $m_{i}$ and can therefore commit at most this amount $m_{i}$ to the project.

Assumption 1 Each agent could afford to pay the costs $c: m_{i} \geq c$ for each $i$.
Assumption 1 limits our focus on projects which are not simply rejected because of their per capita costs exceeding individual budget constraints.

### 2.1.2 Heterogenous valuations under uncertainty

We study the case of a project $\pi$ with an uncertain benefit which is the same for all agents. We model the benefit as a random variable $Y$ with the state space $[\underline{y}, \bar{y}]$, $0 \leq \underline{y}<\bar{y}<\infty$, with $\underline{y}$ and $\bar{y}$ being common knowledge. ${ }^{2}$ We are interested in projects which are too costly for a single agent to be realized on his own but may be expected to be beneficial:

Assumption 2 The costs of project $\pi$ are such that $\underline{y}<c<\bar{y}<C$.

[^0]We suppose that each agent forms his decision to vote for or against a project by maximizing his utility function subject to his budget constraint: We denote agent $i$ 's utility function with $V_{i}(\cdot)$, which is a function mapping non-negative random variables into real numbers. We assume that $V_{i}(\cdot)$ is only privately known to each agent and can be represented as a von Neumann-Morgenstern utility function. Therefore we suppose $V_{i}(X)=E_{i} v_{i}(X)$, with $v_{i}^{\prime}(x)>0, v_{i}^{\prime \prime}(x)<0$, to reflect risk aversion or $v_{i}(x)=x$ in the case of risk neutrality.

An agent $i$ will vote for (against) project $\pi$ if $V_{i}\left(m_{i}+Y-c\right) \geq(<) V_{i}\left(m_{i}\right)$. We will stretch the notation of project $\pi$ a little bit and write $V_{i}\left(m_{i}+Y-c\right)$ short as $V_{i}(\pi)$, and $V_{i}\left(m_{i}\right)$ short as $V_{i}(\not \approx)$. Consequently, agent $i$ is a supporter of $\pi$ if $V_{i}(\pi) \geq V_{i}(\not \not)$, and an opposer of $\pi$ if $V_{i}(\pi)<V_{i}(\nRightarrow)$.

Assumption 3 Before the project is realized, i.e., at the time of the vote for or against project $\pi$, agents differ in their expected utility of $\pi, V_{i}(\pi)$.

There are two intuitive applications which support heterogenous $V_{i}(\cdot)$ across the agents: Different levels of risk aversion and different beliefs.

## Heterogeneity in risk aversion

Agents may agree on the distribution of $Y$, but have different levels of risk aversion. This means that the expected utility of $\pi$ equals

$$
V_{i}(\pi)=E\left[v_{i}\left(m_{i}+Y-c\right)\right] .
$$

Let us consider an example. For simplicity, we look at two agents for which $m_{i}=m_{j}=m$ and $V_{i}(m)=V_{j}(m)$. Agents differ in their utility functions $v_{i}(x)=x^{\alpha_{i}}$ and $v_{j}(x)=x^{\alpha_{j}}, x \geq 0$, with $0 \leq \alpha_{i}<\alpha_{j} \leq 1$ implying that agent $i$ is more risk averse than agent $j$. Consider the case that $Y$ is a binary random variable, taking the values $\bar{y}>0$ with probability $p$ and $\underline{y}>0$ with probability $1-p$, such that both agents expect the payoff

$$
\begin{equation*}
(1-p)(m-c+\underline{y})+p(m-c+\bar{y})>m \tag{1}
\end{equation*}
$$

when $\pi$ is implemented. In Figure 1 we show the utility functions of agents $i$ and $j$. Despite of (1) agent $i$ votes against $\pi$ since $V_{i}(\not \not \subset)>V_{i}(\pi)$, while agent $j$ votes for $\pi$ because of $V_{j}(\pi)>V_{j}(\nRightarrow)$.


Figure 1: Heterogenous levels of risk aversion.

Note that even if all agents have the same utility function, $v_{i}(\cdot)=v(\cdot)$, having different initial endowments $m_{i}$ can lead to heterogenous levels of risk aversion in the same way that different utility functions do. We present such an example in Section 2.2.4.

## Heterogeneity in beliefs about the value of the project

Agents may have different individual beliefs about $\pi$ 's benefit, i.e., the distribution of $Y$. The differences in beliefs may have several origins.

Let us first suppose that agents share a common prior belief about the distribution of $Y$ and receive private information which can be used to update their beliefs. There are several reasons why they might fail to harmonize on their posterior beliefs (Aumann, 1976). One reason is the lack of incentive to share private information. Specifically, if agents have different levels of risk aversion, those who are more risk averse have an incentive to retain optimistic information from the others and vice versa. Other reasons are that large populations face high transaction costs when exchanging private information, or that agents are boundedly rational (see e.g. Hanson, 2003) and do not make use of information optimally.

Moreover, agents may not share a common prior belief (Morris, 1995), and therefore will not reach common expectations even if they share all private information and have the same public information available. ${ }^{3}$

For heterogenous beliefs, the expected utility of $\pi$ equals

$$
V_{i}(\pi)=E_{i}\left[v_{i}\left(m_{i}+Y-c\right)\right]
$$

where the operator $E_{i}[\cdot]$ refers to heterogenous beliefs after all possible belief updates have been made.

### 2.1.3 Financing the public good

If the majority of agents are supporters of $\pi$, the project will be implemented after the population votes on it. Each agent bears the costs $c$ and gets the benefit $y_{\pi}$, which is the realization of the random variable $Y$.

The procedure we suggest in Section 2.2 addresses the case in which the project is not implemented after the vote. In such a setup, there exists a potential freerider problem. Define

$$
\begin{equation*}
\bar{c}_{i}=\sup \left\{0 \leq c_{i} \leq m_{i}: V_{i}\left(m_{i}+Y-c_{i}\right)>V_{i}\left(m_{i}\right)\right\}, \tag{2}
\end{equation*}
$$

i.e., $\bar{c}_{i}$ is the maximum amount that agent $i$ would be willing to pay for project $\pi$. In case

$$
\begin{equation*}
\sum_{i=1}^{N} \bar{c}_{i}>C \tag{3}
\end{equation*}
$$

the project could in principle be financed by voluntary contributions, with agent $i$ not contributing more than $\bar{c}_{i}$. But since the utility function $V_{i}(\cdot)$ is private information, the probability that the agents provide the costs for the project voluntarily is strictly smaller than one (Menezes et al., 2001). In fact, it goes to zero for large populations (Mailath and Postlewaite, 1990). ${ }^{4}$ Gradstein (1994)

[^1]presents a revelation mechanism for the case of privately known costs associated with the agent's participation in the provision of a discrete public good. His suggested mechanism is incentive compatible and implements the public good with probability one. However, when the outside option to the mechanism is voluntary contribution, the mechanism is only individually rational under very special assumptions, i.e., if the cost distribution is uniform.

### 2.2 The contingent claim

We now define a three-step procedure $P$, which might result in the creation of a contingent claim that finances the costs of $\pi$.

### 2.2.1 The procedure

1. The authority organizes a vote and assesses the majority for or against $\pi$ under the following two conditions:
(a) The costs for realizing $\pi$ are paid by the revenue from the disposition of contingent claims issued by the authority. The features of the contingent claims are as described in steps 2 and 3 .
(b) From the project's benefit $y_{\pi}$, with $y_{\pi}$ being the realized value of $Y$, every agent has to transfer the share $\tau y_{\pi}$ to the authority, $\tau \in(0,1)$.

If a majority of agents supports $\pi$ under the conditions (a) and (b), the procedure moves to step 2 . If a majority rejects $\pi$, the authority closes the case of project $\pi$, and takes no further actions.
2. The authority announces a period of time during which it sells the contingent claim on a first-come-first-serve basis ${ }^{5}$ and the conditions of the claim: The price of one claim is $c$, there are $N$ claims available, and one contingent claim yields the payoff $\tau y_{\pi}$.

[^2]If all $N$ claims are sold, the procedure moves to step 3 . Otherwise, the authority repays the buyers what they have paid for the claims, closes the case of project $\pi$, and takes no further action.
3. Project $\pi$ is implemented, the authority covers the costs $C$ with the revenues from the sale of the contingent claims. Once the project is implemented, the authority collects the transfer $\tau y_{\pi}$ from each agent and uses the proceeds to pay off the claim holders. Note that the authority has a balanced budget, i.e., does not make a gain or a loss from the revenues and expenses arising from the project and the contingent claims.

Figure 1 illustrates procedure $P$, the notation is explained in the next section.


### 2.2.2 The equilibrium

We now provide a condition under which the procedure reaches step 3, i.e., under which project $\pi$ is implemented.

Under our claim solution, when $\tilde{K}$ claims are sold, and agent $i$ buys $K_{i}$ claims,
his utility becomes

$$
V_{i}\left(\pi \mid \tilde{K}, K_{i}\right)=\left\{\begin{array}{cl}
V_{i}\left(m_{i}+Y(1-\tau)+K_{i}(\tau Y-c)\right) & \text { if } \tilde{K}=N \\
V_{i}\left(m_{i}\right) & \text { if } \tilde{K}<N
\end{array}\right.
$$

The function $V_{i}(\pi \mid N, \cdot)$ has the following properties.

1. Concavity:

$$
\frac{\partial^{2} V_{i}\left(\pi \mid N, K_{i}\right)}{\partial K_{i}^{2}}=E_{i}\left[(\tau Y-c)^{2} v_{i}^{\prime \prime}\left(m_{i}+Y(1-\tau)+K_{i}(\tau Y-c)\right)\right] \leq 0
$$

with strict inequality, unless $Y$ is degenerate or in case of risk neutrality $\left(v_{i}(x)=x\right)$.
2. Comparison:

$$
V_{i}(\pi \mid N, 0)=V_{i}\left(m_{i}+Y(1-\tau)\right) \geq V_{i}\left(m_{i}\right)
$$

with strict inequality, unless $\tau=1$, or $Y$ is degenerate and takes value 0 .
We define

$$
\bar{K}_{i}=\sup \left\{0 \leq K \leq \frac{m_{i}}{c}: V_{i}(\pi \mid N, K) \geq V_{i}\left(\pi \mid N, K^{\prime}\right), 0 \leq K^{\prime} \leq K\right\}
$$

In case $V_{i}\left(\pi \mid N, K^{\prime}\right)<V_{i}\left(m_{i}\right)$ for some $K^{\prime} \in\left[0, \frac{m_{i}}{c}\right]$, we also define

$$
\hat{K}_{i}=\inf \left\{0 \leq K \leq \frac{m_{i}}{c}: V_{i}\left(\pi \mid N, K^{\prime}\right)<V_{i}\left(m_{i}\right), K^{\prime}>K\right\} .
$$

Figure 2 shows typical examples of the graph of $V_{i}(\pi \mid N, \cdot)$.
Proposition 1 Elimination of dominated strategies implies the following behavior. For every tax rate $\tau$, all agents vote in favor of implementing $\pi$ under the conditions in step 1. In step 2 each agent $i$ orders at least $\bar{K}_{i}$ claims. Therefore, if $\tau$ is such that

$$
\begin{equation*}
\sum_{i=1}^{N} \bar{K}_{i} \geq N \tag{4}
\end{equation*}
$$

step 3 is reached and project $\pi$ is realized.


Figure 2: Four different cases of $V_{i}(\pi \mid N, \cdot)$. Bottom right: Risk neutral utility function.

Proposition 1 states that the strategy of ordering $\bar{K}_{i}$ or more claims in step 2 dominates all strategies of ordering less claims. We focus on dominant strategy equilibria because they are more robust than the Bayesian-Nash equilibria: We do not need to make assumptions on the information available to agents about other agents' preferences or rationality.

Proof. We argue first that if the procedure $P$ reaches step 2, agent $i$ 's strategy of ordering $\bar{K}_{i}$ claims dominates any alternative strategy of ordering $K_{i}$ claims if $K_{i}<\bar{K}_{i}$.

When agent $i$ orders $\bar{K}_{i}$ claims, the outcome depends on the number of claims the other agents order. There are several cases to consider.
A) If agent $i$ receives $\bar{K}_{i}$ claims from the authority, it means that all $N$ claims are sold. In this case agent $i$ 's utility is $V_{i}\left(\pi \mid N, \bar{K}_{i}\right)$, which is optimal.
B) If agent $i$ receives 0 claims from the authority, it may be for two reasons:

B1) When agent $i$ orders the claims, $N$ orders have already been made. In this case, his utility is $V_{i}(\pi \mid N, 0)$. If he would order fewer claims, his utility would be the same.

B2) After the selling period ends, less than $N$ claims have been ordered. In this case, agent $i$ 's utility is $V\left(m_{i}\right)$. If he ordered fewer claims, the outcome would have been the same.
C) If agent $i$ receives $K_{i}^{\prime} \in\left(0, \bar{K}_{i}\right)$ claims from the authority, it means that when he ordered $\bar{K}_{i}$ claims, exactly $N-K_{i}^{\prime}$ claims had already been ordered and all $N$ claims are sold after his order. His utility is $V_{i}\left(\pi \mid N, K_{i}^{\prime}\right)$. There are two cases to consider.

C1) If agent $i$ ordered $K_{i} \in\left[K_{i}^{\prime}, \bar{K}_{i}\right)$ claims, the outcome would be the same:
He would receive $K_{i}^{\prime}$ claims, all $N$ claims would be sold and his utility would be $V_{i}\left(\pi \mid N, K_{i}^{\prime}\right)$.
$\mathrm{C} 2)$ If agent $i$ ordered $K_{i} \in\left[0, K_{i}^{\prime}\right)$ claims, there are two cases to consider:
C2i) Total number of orders turns out to be less than $N$. In this case agent $i$ 's utility is $V_{i}\left(m_{i}\right) \leq V_{i}(\pi \mid N, 0) \leq V_{i}\left(\pi \mid N, K_{i}^{\prime}\right)$, since $0 \leq K_{i}^{\prime} \leq \bar{K}_{i}$.
C2ii) Total number of orders is at least $N$. Agent $i$ 's utility becomes $V_{i}\left(\pi \mid N, K_{i}\right) \leq V_{i}\left(\pi \mid N, K_{i}^{\prime}\right)$, since $0 \leq K_{i}<K_{i}^{\prime} \leq \bar{K}_{i}$.

It is clear that all agents vote yes for $\pi$ under $P$, since they have the option in step 2 to order no claims and obtain then utility $V_{i}(\pi \mid N, 0) \geq V_{i}\left(m_{i}\right)$ or $V_{i}\left(m_{i}\right)$, depending on all claims being sold or not.

It is interesting to observe that an agent $i$ may want to order more than $\bar{K}_{i}$ claims. This may happen if he believes that by doing so, he can change the total of sold claims from a number smaller than $N$ to $N$. Therefore, the project could be implemented even if (4) fails.

However, in step 2, ordering $\hat{K}_{i}$ claims dominates ordering any $K_{i}>\hat{K}_{i} .{ }^{6}$

[^3]Therefore no agent will in any case order more than $\hat{K}_{i}$ claims. This is used in Proposition 2.

Proposition 2 If $E_{i}[Y]<c$, then $\hat{K}_{i}<1$. So if agents have common expectations with $E[Y]<c$, then $\sum \hat{K}_{i}<N$ and $\pi$ is not implemented.

Proposition 2 states that if agents have common expectations, the procedure only implements ex ante socially beneficial projects, e.g., projects which satisfy $E[Y]>c$.

Proof. We have

$$
V_{i}(\pi \mid N, 1)=V_{i}\left(m_{i}+Y-c\right)<V_{i}\left(m_{i}\right),
$$

where the inequality is a consequence of $E_{i}[Y]<c$, by a well known result about risk-averse utility functions (see e.g. Gollier, 2004, Prop. 6, p. 54). Since $V_{i}(\pi \mid N, 0) \geq V_{i}\left(m_{i}\right)$ and $V_{i}(\pi \mid N, \cdot)$ is concave, the display above implies $V_{i}\left(\pi \mid N, K_{i}\right)<V_{i}\left(m_{i}\right)$ for all $K_{i} \geq 1$ and hence $\hat{K}_{i}<1$ (note that $\hat{K}_{i}$ is defined in this case, since $\frac{m_{i}}{c} \geq 1$ by assumption 1.)

If an agent buys exactly one claim, his utility from the claim solution is

$$
V_{i}(\pi \mid N, 1)=V_{i}\left(m_{i}+Y-c\right),
$$

when all claims are sold. This is equal to the utility he would have had if project $\pi$ had been accepted by majority vote. This observation implies that with a somewhat different selling mechanism of the claims, the pre-claim voting stage can be neglected. An example of such a selling mechanism is that agents can only order a minimum of one claim and each agent who ordered claims receives one claim before the remaining claims are assigned as above. Note that even if a project could pass the majority vote, all agents are weakly better off with the claim solution.

Note that procedure $P$ does not implement every project $\pi$ which is ex ante socially beneficial. For some $\pi$, although $E[Y]>c$, the number of claims bought can be smaller than $N$. It could be the case that some agents, by buying $K_{i}$ with $\bar{K}_{i}<K_{i}<\hat{K}_{i}$ claims, could increase the number of claims sold to $N$. This would allow for $\pi$ to be implemented which would increase their utility. However, since

$$
V_{i}\left(\pi \mid N, \bar{K}_{i}\right)>V\left(\pi \mid N, K_{i}\right)>V\left(\pi \mid N, \hat{K}_{i}\right) \geq V(\not \not),
$$

buying more than $\bar{K}_{i}$ claims is equivalent to a voluntary contribution for the provision of the public good and we face a traditional free-rider problem. We refer to the respective literature for solutions to this problem.

### 2.2.3 A secondary market for claims

The analysis above has abstracted from the possibility that after the claims are sold to the public a secondary private market for them could be created. The anticipation that such a market will be created could increase agent $i$ 's demand for claims to more than $\bar{K}_{i}$. This is the case if he believes that some agents who value the claims more than he does cannot buy their optimal number of claims due to the random order agents get to place their orders. See Harrison and Kreps (1978) and Morris (1996) for analysis of such considerations.

Note that Proposition 1 and the statement in Proposition 2 for common expectations are not affected by this observation.

### 2.2.4 An example with identical log-utility functions

We now provide an example in order to demonstrate the working of the suggested claim solution. All agents have the utility function $v_{i}(x)=v(x)=\ln (x)$ and heterogenous initial endowments of either $m_{i}=m_{p}$ (poor) or $m_{i}=m_{r}$ (rich) with $m_{p}<m_{r}$. We consider a project $\pi$ which benefit is a binary random variable, taking the value $\underline{y}$ with probability $p$ and $\bar{y}$ otherwise.

We assume the following values for the variables: $m_{p}=1, m_{r}=20, \underline{y}=0$, $\bar{y}=4, p=0.85, c=0.25$. There are more poor than rich agents.

Under majority rule, project $\pi$ is rejected. The reason is that the poor agents are better off without the project if the costs are shared:

$$
\begin{aligned}
V_{p}(\pi) & =p \ln \left(m_{p}-c+\underline{y}\right)+(1-p) \ln \left(m_{p}-c+\bar{y}\right)=-0.01<0 \\
V_{p}(\not \not \subset) & =\ln \left(m_{p}\right)=0
\end{aligned}
$$

However, all agents are in favor of financing $\pi$ with the claims. The utility function of the rich,

$$
\begin{aligned}
V_{r}(\pi \mid N, K)= & p \ln \left(m+\underline{y}(1-\tau)+K_{r}(\tau \underline{y}-c)\right) \\
& +(1-p) \ln \left(m+\bar{y}(1-\tau)+K_{r}(\tau \bar{y}-c)\right),
\end{aligned}
$$

is maximized at $K^{*}(\tau)$. We find that $K^{*}(0.5)=1.31, K^{*}(0.6)=3.46, K^{*}(0.7)=$ $4.93, K^{*}(0.8)=6.01$. So if we have $\tau=0.6$ and not more than two thirds of the population are poor, then the rich order enough claims to finance $\pi$.

The expected utility of the poor increases from $V_{p}(\not \not)=0$ to $V_{p}(\pi \mid N, 0)=$ 0.14 when $\pi$ is financed by the claim and $\tau=0.6$.

This example demonstrates that even if the heterogeneity in risk aversion can solely be attributed to heterogeneity in wealth as opposed to differences in utility functions, the claim solution is beneficial for all agents.

### 2.2.5 Interpreting the transfer share

If $c<\tau \bar{y}$, then $\bar{K}_{i}=0$. Therefore (4) can only be possibly true if $\tau \geq \frac{c}{\bar{y}}$. The authority has a degree of freedom in choosing $\frac{c}{\bar{y}} \leq \tau \leq 1$. The larger $\tau$ is, the more likely it is that (4) will hold.

The transfer share $\tau$ is a measure of how much the claim holders as opposed to the non-claim holders gain from the claim solution. If $\tau$ is equal to its lower bound, $\tau=\frac{c}{y}$, the claim holders cannot make a profit from holding the claim, but may benefit from the project $\pi$. If $\tau$ is equal to its upper bound, $\tau=1$, those who do not hold a claim have to transfer the whole benefit from $\pi$ to the authority.

Some applications of the model may require $\tau$ to be exogenous. For instance, $\tau$ could be the tax rate at which income is taxed in a society, so any additional income from the project would be taxed at the same tax rate.

When the value of $\tau$ is not predetermined in some exogenous way, one may want to optimize its choice under some criterion, or alternatively, sell the claims using some mechanism which effectively endogenizes $\tau$. One may look for a mechanism that optimizes efficiency in the model, with the claims being sold to those who value them most. But since this kind of efficiency can always be implemented by a secondary market for claims after the authority has sold them to the public, this is not a major criterium to be used (if transaction costs are not considered). Note also that one may take other issues in consideration when deciding how to sell the claims, as in the last paragraph in Section 2.2.2.

### 2.2.6 Changing the constitution

Will a population add procedure $P$ to its institutions? Or put differently, is $P$ individually rational? The following amendment is proposed to the constitution: Whenever a project $\pi$ with the properties defined in Section 2.1 is rejected by majority vote, every agent can ask the authority to start procedure $P$ for this project.

This amendment will be approved unanimously. As explained in Section 2.2.2, supporters as well as opposers ex ante benefit from the claim solution. Consequently, no agent will vote against the amendment.

## 3 Conclusions

The claim solution allows supporters of a project to win over the votes of those who initially opposed it because they are risk averse or expect the project to be socially inefficient. The claim contingent on the value of the project shifts the project's risk from those who are not willing to take it to those who are. The suggested procedure makes all agents ex ante better off and should therefore have good chances to be added to the set of institutional rules of a population. The creation of contingent claims related to the taxable income of a society has also advantages to agents outside of this society, who can use such claims to diversify their investments (Shiller, 2003).

It is very likely that people differ in their levels of risk aversion or that there is disagreement about the value of unacquainted and innovative projects. The claim solution is an option on how to complement majority rule for decisions in groups. Much more work is needed to apply the idea to a political economy setting. Is it possible that visionaries who believe to know welfare enhancing strategies can insure sceptics such that politics become more dynamic and innovative? We hope that this research motivates work in this direction.

Many theoretical and practical questions have to be solved for such applications. Among other things, the final value of the public good may be either private and unobservable, or very difficult to measure. These are challenging but exciting problems to solve.

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[^0]:    ${ }^{1}$ For an interesting discussion of referendum mechanisms and their advantages see Ledyard and Palfrey (2002).
    ${ }^{2}$ The existence of a finite upper bound $\bar{y}$ is not crucial for our results but makes the analysis simpler. The assumption that $\underline{y}>0$ can be replaced with the assumption that $\underline{y}>-\infty$; this case reduces to the one discussed in this paper by redefining $c$ to include $|\underline{y}|$ when $\underline{y}<0$.

[^1]:    ${ }^{3}$ For research foregoing the common prior assumption, see, e.g., Yildiz (2003), Fisher (2005), or Billot et al. (2002).
    ${ }^{4}$ Palfrey and Rosenthal (1984, 1988); Bagnoli and Lipman (1989, 1992) study voluntary provision of discrete public goods under complete information. Nitzan and Romano (1990) introduce uncertainty about costs which can result in a unique un-dominated but inefficient equilibrium, however McBride (2005) shows that the relationship between the degree of threshold uncertainty and equilibrium contributions is not monotonic.

[^2]:    ${ }^{5}$ Other ways to sell the claims will be discussed later. For mathematical precision, we can think that the agents are ordered in some random fashion and then, in this order, buy one at a time as many claims as they demand capped by the amount not sold yet.

[^3]:    ${ }^{6}$ This can be proved in a way similar to that used for Proposition 1.

