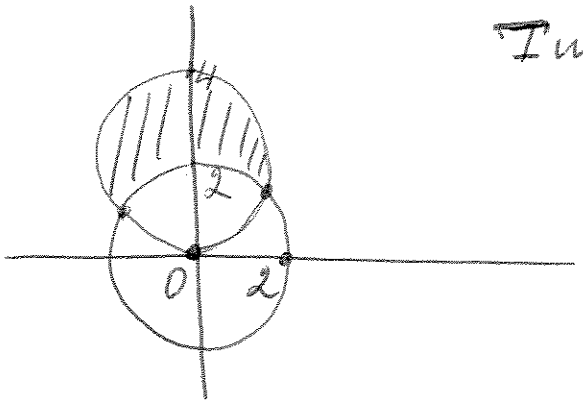
Quiz 2. (Tuesday).

(1). Find the area of the region inside of the circle $r = 4\sin\theta$ and outside of the circle $r = 2$.



Two circles intersect at two points:

$$4\sin\theta = 2 \iff \sin\theta = \frac{1}{2}$$

$$\iff \theta = \pi/6 \text{ and } \theta = 5/6\,\pi.$$

$$\text{Area} = \int_{\pi/6}^{5/6\pi}\int_{2}^{4\sin\theta} r\,dr\,d\theta =$$

$$= \int_{\pi/6}^{5/6\pi} \frac{r^2}{2}\Big|_2^{4\sin\theta} d\theta = \frac{1}{2}\int_{\pi/6}^{5/6\pi} (16\sin^2\theta - 4)\,d\theta ;$$

$$\int \sin^2\theta\,d\theta = \int \frac{1-\cos 2\theta}{2}\,d\theta = \frac{\theta}{2} - \frac{1}{4}\sin 2\theta.$$

$$\text{Area} = \frac{1}{2}\cdot\left(16\cdot\left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)\Big|_{\pi/6}^{5/6\pi} - 4\theta\Big|_{\pi/6}^{5/6\pi}\right) =$$

$$= \frac{1}{2}\left(8\cdot\frac{2}{3}\pi - 4\cdot\left(-\frac{\sqrt{3}}{2}\cdot 2\right) - 4\cdot\frac{2}{3}\pi\right) = \frac{1}{2}\left(\frac{4}{3}\pi + \frac{4\sqrt{3}}{2}\right) =$$

$$= \frac{4}{3}\pi + 2\sqrt{3}.$$

(2). For a plate that occupies the region

$$D = \{(x,y) \mid \begin{matrix} 0 \le x \le a \\ 0 \le y \le b \end{matrix}\}$$ with density

$$\rho(x,y) = xy,$$

find the moment M_x with respect to x-axis.

$$M_x = \iint_D y\,\rho(x,y)\,dx\,dy = \int_0^a \int_0^b y \cdot xy \, dy\,dx =$$

$$= \int_0^a \int_0^b xy^2\,dy\,dx = \frac{x^2}{2}\Big|_0^a \cdot \frac{y^3}{3}\Big|_0^b = \frac{a^2 b^3}{6}$$