

Quiz 2. (Thursday).

① Find the volume of the solid bounded by the paraboloid

$$z = 10 - 3x^2 - 3y^2$$

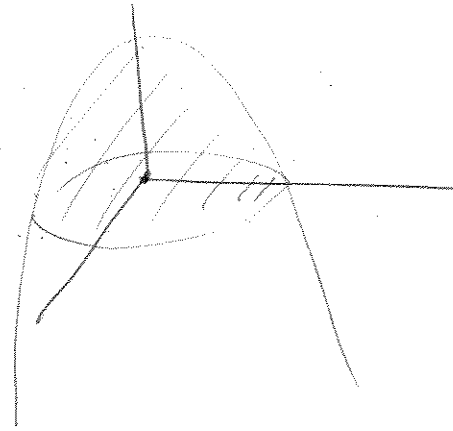
and the plane z = 4.

$$10 - 3x^2 - 3y^2 = 4 \quad \Leftrightarrow \quad 3(x^2 + y^2) = 6$$
$$x^2 + y^2 = 2.$$

Region of integration:

$$D = \{(x, y) \mid x^2 + y^2 \leq 2\} =$$

$$= \{(r, \theta) \mid r^2 \leq 2\}.$$



$$\text{Volume} = \int_0^{2\pi} \int_0^{\sqrt{2}} ((10 - r^2 \cdot 3) - 4) r dr d\theta =$$

$$= 2\pi \cdot \left( \frac{10}{2} r^2 - 3 \frac{r^4}{4} \right) \Big|_0^{\sqrt{2}} =$$

$$= 2\pi \cdot (10 \cdot 1 - 3 \cdot \frac{4}{4}) = \cancel{2\pi \cdot 7} \quad \boxed{6\pi}$$

2. Find the moment of inertia  $I_x$  (with respect to x-axis) of the flat plate

$$D = \left\{ (x, y) \mid \begin{array}{l} 0 \leq x \leq 4 \\ 0 \leq y \leq \pi/2 \end{array} \right\}$$

with the density  $f(x, y) = \cos y$

$$I_x = \int_0^4 \int_0^{\pi/2} y^2 \cos y \, dy \, dx$$

$$\int y^2 \cos y \, dy = y^2 \sin y - \int \sin y \cdot 2y \, dy$$

$$\boxed{\begin{array}{l} u = y^2; \quad du = 2y \, dy; \\ dv = \cos y \, dy; \quad v = \sin y \end{array}}$$

$$\boxed{\begin{array}{l} u = y, \quad du = dy \\ dv = \sin y; \quad v = -\cos y \end{array}}$$

$$= y^2 \sin y - 2 \left( -y \cos y + \int \cos y \, dy \right) =$$

$$= y^2 \sin y + 2y \cos y - 2 \sin y + C.$$

$$I_x = \int_0^4 \left( y^2 \sin y + 2y \cos y - 2 \sin y \right) \Big|_0^{\pi/2} dx =$$

$$= \int_0^4 \left( \frac{\pi^2}{4} + \pi \cdot 0 - 2 - 0 \right) dx = 4 \cdot \left( \frac{\pi^2}{4} - 2 \right) = \pi^2 - 8.$$