MATH 32B
FIRST MIDTERM EXAMINATION

Solutions

October 27th, 2006

Please show your work. You will receive little or no credit for a correct answer to a problem which is not accompanied by sufficient explanations. If you have a question about any particular problem, please raise your hand and one of the proctors will come and talk to you. At the completion of the exam, please hand the exam booklet to your TA. If you have any questions about the grading of the exam, please see the instructor *within 15 calendar days of the examination.*

Name:____________________ Section:____________________

#1	#2	#3	#4	#5		Total

Problem 1. Compute the integral $I = \int\int_R \frac{xy^2}{x^2+1} dA$, where $R = [0,1] \times [-3,3]$.

$$I = \int_0^1 \int_{-3}^{3} \frac{x}{x^2+1} y^2 dy\, dx =$$

$$= \int_0^1 \frac{x\,dx}{x^2+1} \cdot \int_{-3}^{3} y^2 dy =$$

$$\boxed{\begin{array}{l|l} u(0)=1 & u = x^2+1 \\ u(1)=2 & du = 2x\,dx \end{array}}$$

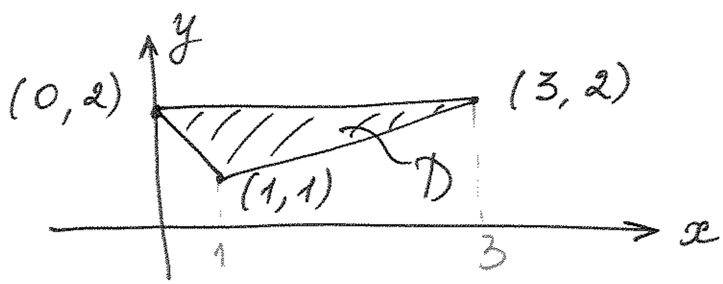
$$= \frac{1}{2}\int_1^2 \frac{du}{u} \cdot \left.\frac{y^3}{3}\right|_{-3}^{3} =$$

$$= \frac{1}{2} \ln u \Big|_1^2 \cdot \left(\frac{27}{3} - \frac{-27}{3}\right) =$$

$$= \frac{1}{2} \ln 2 \cdot 9 \cdot 2 = 9 \ln 2$$

2
2
2

Problem 2. Integrate the function $f(x, y) = y$ over the triangle with the vertices $(0, 2)$, $(1, 1)$, $(3,2)$.



Or

$$\int_0^1 \int_{2-x}^{2} y \, dy \, dx + \int_1^3 \int_{\frac{x+1}{2}}^{2} y \, dy \, dx$$

$$D = \{ (x,y) \mid 1 \le y \le 2, \quad 2-y \le x \le 2y-1 \}$$

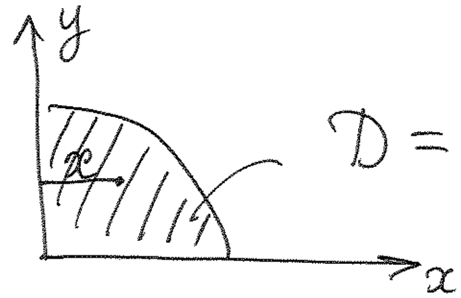
$$I = \int_1^2 y \int_{2-y}^{2y-1} dx \, dy = \int_1^2 y \cdot x \Big|_{2-y}^{2y-1} dy =$$

$$= \int_1^2 y \cdot (2y-1-2+y) \, dy = \underline{\int_1^2 y(3y-3) \, dy} =$$

$$= 3 \int_1^2 (y^2 - y) \, dy = \left(y^3 - \frac{3}{2} y^2 \right) \Big|_1^2 =$$

$$= \left(8 - \frac{3}{\cancel{2}} \cdot \cancel{4}^{2} \right) - \left(1 - \frac{3}{2} \right) = 2 - \left(-\frac{1}{2} \right) = 2.5$$

Problem 3. A lamina (flat plate) occupies the part of the disc $x^2 + y^2 \le 1$ that lies in the first quadrant. Its density at any point is proportional to its disance from the x-axis: $\rho(x, y) = cy$, where c is a given constant. Find the moment M_y of this lamina with respect to the y-axis.



$$D = \{(r,\theta) \mid 0 \le r \le 1;\ 0 \le \theta \le \pi/2\}.$$

3

$$M_y = \iint_D x \cdot \rho(x,y)\,dA = \int_0^{\pi/2}\int_0^1 r\cos\theta \cdot c\,r\sin\theta \cdot r\,dr\,d\theta =$$

Pons: 2+3

$$= c\int_0^{\pi/2} \cos\theta\cdot\sin\theta\,d\theta \cdot \int_0^1 r^3\,dr = c\int_0^{\pi/2} \frac{\sin 2\theta}{2}\,d\theta \cdot \frac{r^4}{4}\Big|_0^1 =$$

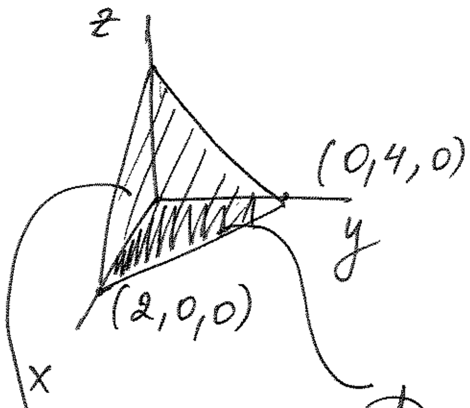
$$= \frac{c}{2}\cdot\frac{1}{4}\cdot\int_0^{\pi} \sin\alpha \cdot \frac{d\alpha}{2} = \frac{c}{16}\cdot\int_0^{\pi} \sin\alpha\,d\alpha = \frac{c}{16}\cdot -\cos\alpha\Big|_0^{\pi} =$$

$\alpha = 2\theta$
$d\alpha = 2\,d\theta$
$\alpha(0) = 0,\ \alpha(\pi/2) = \pi$

$$= \frac{c}{16}\cdot 2 = \frac{c}{8}.$$

Int = 2.

Problem 4. Write down (but don't compute!) the triple integral of the form $\int_a^b \int_c^d \int_e^f dz dx dy$ representing the volume of the tetrahedron enclosed by the coordinate planes and the plane $2x + y + z = 4$.



$$z = 4 - 2x - y = f(x,y)$$

$$D = \left\{ (x,y) \;\middle|\; 0 \le x \le \frac{4-y}{2} = 2 - y/2 \right\}$$

$$0 \le y \le 4$$

$$E = \{ (x,y,z) \mid 0 \le x \le 2 - y/2,\ 0 \le y \le 4,\ 0 \le z \le 4 - 2x - y \}$$

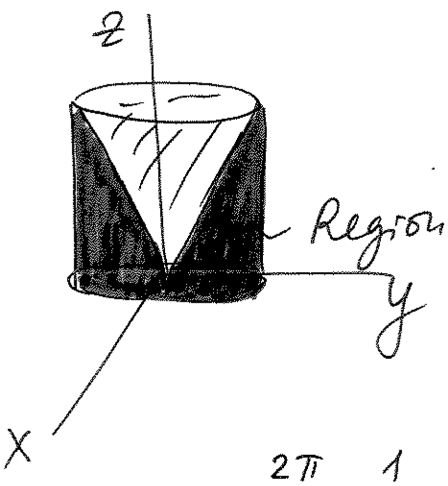
$$\text{Volume} = \iint_D f(x,y)\, dA = \iiint_E 1 \cdot dV$$

$$\left(= \int_0^4 \int_0^{2-y/2} (4 - 2x - y)\, dx\, dy \right)$$

$$\boxed{\text{Volume} = \int_0^4 \int_0^{2-y/2} \int_0^{4-2x-y} dz\, dx\, dy}$$

Problem 5. Integrate the function $f(x,y,z) = \frac{z}{\sqrt{x^2+y^2}}$ over the region that lies below the surface $z^2 = x^2 + y^2$, above the xy-plane, and inside of the cylinder $x^2 + y^2 = 1$.

Region: $E = \left\{ (r,\theta,z) \;\middle|\; \begin{array}{l} 0 \le r \le 1 \\ 0 \le z \le r \\ 0 \le \theta \le 2\pi \end{array} \right\}$



Region of integration.

$f = \frac{z}{r}$.

$$I = \int_0^{2\pi}\int_0^1\int_0^r \frac{z}{\cancel{r}} \cdot \cancel{r}\, dz\, dr\, d\theta =$$

$$= 2\pi \int_0^1 \int_0^r z\, dz\, dr = \cancel{2}\pi \int_0^1 \frac{r^2}{\cancel{2}}\, dr = \pi \cdot \frac{r^3}{3}\Big|_0^1 = \frac{\pi}{3}.$$