PRACTICE PROBLEMS FOR MATH 167 (MIDTERM)

These problems are intended to give you an opportunity to prepare for the midterm. The problems on the actual exams will be of a similar difficulty. Please, also review all homework problems, quiz problems, and examples considered in class.

Topics covered: 2-player games of perfect information with no chance moves; Game trees; strategic form of the game; Zermelos' Theorem; Zermelos' Algorithm; The value of the game; Saddle points; Nash equilibrium; subgame-perfect equilibrium; non-competitive (cooperative) games, security strategies.

Basic Probability Theory; Lotteries; Games with chance moves (such as Duel, Parcheesi, and the games considered in homework).

Properties of preference relations; Utility functions; optimizing utility; von Neumann-Morgenstern utility function; risk-loving, risk-neutral and risk-averse behavior.

Payoffs and bimatrix games (up to and including the material on Wednesday's lecture). Friday's lecture is not going to be covered on the exam.

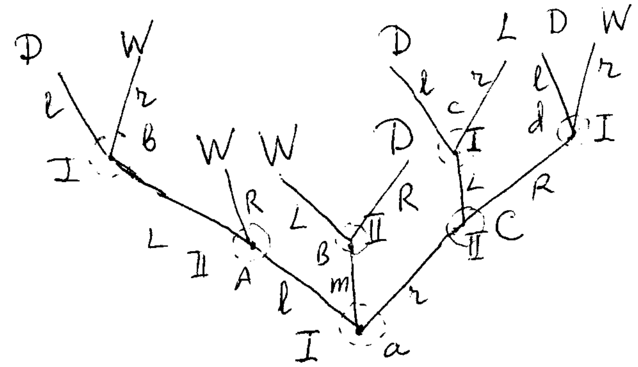
Problem 1. True or False: For each of the following statements, indicate whether it is true or false. (No explanations are required). To discourage random guessing, you will receive +N for a correct answer, 0 if you give no answer, and -N if your guess is wrong.

1. Let $\Omega = \{L, D_1, D_2, D_3, W\}$ be a set of outcomes in a finite 2-player game of perfect information with no chance moves. Assume that we know that Player I can not force an outcome in the set $T = \{D_2, D_3, W\}$. Does it follow that player II can force an outcome D_1?
2. Is it true that the Nash equilibrium determined by the Zermelo's algorithm is necessarily subgame-perfect?
3. Let $E, F \subset \Omega$ be two subsets in the set of all outcomes. Let $p : \Omega \to \mathbb{R}$ be the probability measure. Is it true that $p(E \times F) = p(E) \times p(F)$?
4. Is it true that if $u : \Omega \to \mathbb{R}$ is a von Neumann-Morgenstern function describing a preference relation on a given set of outcomes, then $3u + 1$ is also a von Neumann-Morgenstern utility function describing the same preference relation?
5. Let Alice and John be two rational people. Let $u(x) = x^2$ be the von Neumann-Morgenstern utility function describing Alice's preferences (utility for money). Let $u(x) = x^4$ be the corresponding function describing John's preferences. Let L be a lottery such that Alice decides to participate in this lottery. Does it follow that John will also decide to participate? (Note: the cost of the ticket is the same for Alice and John).

Problem 2. Definitions:

1. Let $\Omega = \{u_1, \dots, u_k\}$ be the set of all outcomes in a 2-player strictly competitive game of perfect information with no chance moves. Let
$$u_1 \prec_1 u_2 \prec_1 \cdots \prec_1 u_k$$
be the preference relation of Player I. What does it mean that $v = u_i$ is the value of the game?
2. What is a Nash equilibrium?
3. Let X be a random variable with possible values $k_1, \dots, k_n$. What is the expectation of X?
4. What is the main property of a von Neumann-Morgenstern utility function?
5. What is a risk-averse behavior?

Problem 3. Let G be a 2-player strictly competitive game with the following game tree:



Draw the strategic form of the game. Find all the Nash equilibria for this game.

Problem 4. Currently, there are two firms making the same product. They are very well established, and unless there is a new company on the market, they have the profits of \$5m and \$1m respectively. The third company wants to start selling a very similar product. Companies I and II decide to pay \$2m and \$0.5m to the third company, if it promises not to enter the market. If this happens, Companies I and II continue to make the same profits as they did before. If the third company decides to enter, company I, being bigger then company II, has the first chance to respond. It can either flood the market with its product, which will result in lowering the profits for everybody (company I gets \$1m, company II gets \$0.2, and company III gets \$0.5m), or split the market, in which case the earnings will be \$3m for company I, \$0.5m for company II, and \$1m for company III. If the market is split, the situation becomes stable, and no other actions come from companies II or III. However, if company I decided to flood the market, company II can either decide to quit business (so, their profits are \$0), but this results in a better situation for both companies I and III: their respective earnings are now \$4m and \$2m. Or, company II may decide to lower the price, which will keep them alive, but will further lower everybody's profits (to \$1m for company I, to \$0.1m for company II and to \$0.3m for company III).

(a) Draw the game tree.
(b) Find the subgame-perfect equilibria.
(c) Find the security levels and security strategies for all players.

Problem 5. A box contains two gold and two silver coins. Two coins are drawn at random. A man looks at the coins that have been drawn, selects one of them and shows it to you. The coin that he shows you is silver. What is the probability that the other coin is also silver?

Problem 6. There are three horses in a race. A bookie offers the odds 2:1 against the first horse, and 3:1 against the second one, and 2:1 against the third one. How should you bet to make money for certain?

Problem 7. A California homeowner has to make a decision as to whether it make sense for him to buy an insurance policy for the next year. The homeowner is assumed to be risk-neutral. The value of his house is \$550.000 (and we assume that it does not change within the year in question). The probability of a devastating earthquake, which will completely demolish the house, is 0.27%. The insurance policy costs \$1450 for this year. Should he buy the policy?

Problem 8. Problem 18, page 162.