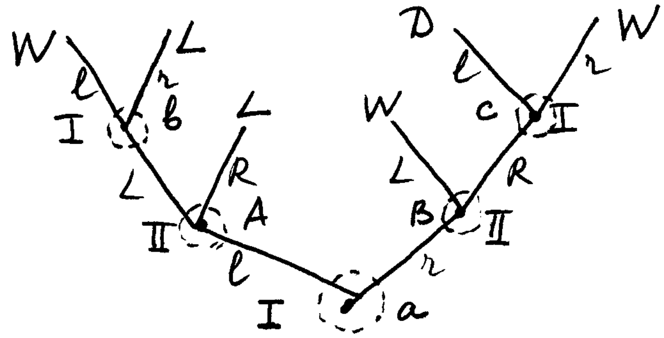
MATH 167
MIDTERM EXAMINATION

Name: Solutions.

Sid:

#1	
#2	
#3	
#4	
#5	
Total	50+10.

Problem 1. The picture below represents the tree of a strictly competitive game with no chance moves.



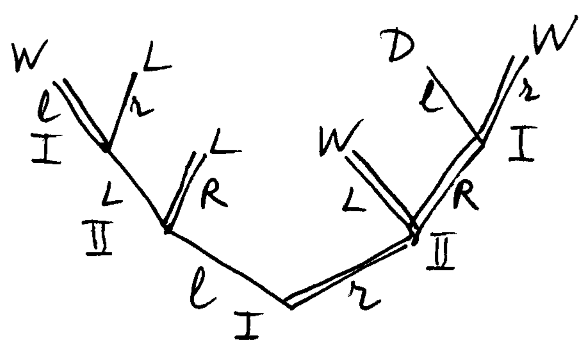
(a) Find the strategic form of the game;
(b) Find all Nash equilibria;
(c) Using Zermelo's algorithm, find all subgame-perfect equilibria.

(a) (5)

I \ II	LL	LR	RL	RR
lll	W	W	L	L
llr	W	W	L	L
lrl	L	L	L	L
lrr	~~W~~ L	~~D~~ L	~~W~~ L	~~D~~ L
rll	W	D	W	D
rlr	W	W	W	W
rrl	W	D	W	D
rrr	W	W	W	W

(b) (5) Nash equilibria: (8) (rlr, XY), (rrr, XY), where $X, Y \in \{R, L\}$

(c) (5)



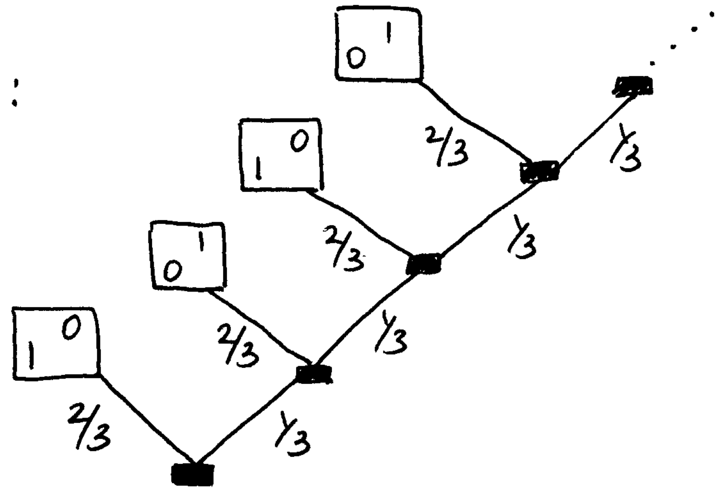
Subgame-perfect equilibria: (2).

(rlr, RX), where $X \in \{L, R\}$

Maximal score: 15.

Problem 2. The following game is being played by two players. First, player I rolls a dice. If the outcome is in the set $\{1, 2, 3, 4\}$, then the payoffs of player I and player II are 1 and 0 respectively. If the outcome is in the set $\{5, 6\}$, it is now player II's turn to roll a dice. This time, if the outcome is in the set $\{1, 2, 3, 4\}$, the resulting payoffs are for player I and player II are 0 and 1 respectively. If the outcome is in the set $\{5, 6\}$, player I rolls the dice, and so on. Assume that both player I and player II are risk-neutral. How much should they be willing to pay to participate in this game?

Game tree:



maximal score:

10

The expected payoff for Player I is

$$\tfrac{2}{3} \cdot 1 + \left(\tfrac{1}{3}\right)^2 \cdot \tfrac{2}{3} + \left(\tfrac{1}{3}\right)^4 \cdot \tfrac{2}{3} + \ldots =$$

$$= \tfrac{2}{3} \cdot \left(1 + \tfrac{1}{9} + \left(\tfrac{1}{9}\right)^2 + \left(\tfrac{1}{9}\right)^3 + \ldots \right)$$

Since $s = 1 + a + a^2 + a^3 + \ldots$ is a geometric series, (with $a = 1/9$)

compute s as follows: $s = 1 + a \cdot (s)$

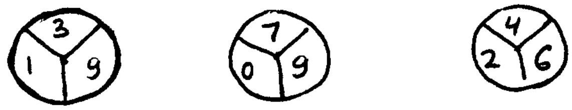
$$\Rightarrow s = \frac{1}{1-a} = \frac{1}{8/9} = 9/8.$$

Thus, the expected payoff for Player I is $\tfrac{2}{3} \cdot \tfrac{9}{8} = \underline{3/4}$. This is how much he should be willing to spend on the ticket for the game. (since he is risk-neutral).

Since the total payoff is always 1, the expected payoff for player II is $1 - 3/4 = \underline{1/4}$. Since he is risk-neutral, this is how much he'll spend.

Problem 3. (A version of Gale's Roulette). Player I begins by choosing a wheel and spinning it. While this wheel is still spinning, player II picks a wheel and also spins it. The player with the larger number wins. The numbers written on the wheels are as follows:

- wheel 1: 1 3 9
- wheel 2: 0 7 8
- wheel 3: 2 4 6



(Making a picture is probably useful).

(a) If Player I chooses wheel 1 and Player II chooses wheel 2, what are the chances that Player I will win the game?

(b) Draw a game tree for this game.

(c) Find a subgame-perfect equilibrium for this game.

(In all the tables below W denotes win for the wheel whose number listed vertically)

(a)

3 ↑ max. score for (a)

Wheel 1 \ Whee 2	0	7	8
1	W	L	L
3	W	L	L
9	W	W	W

The probability that Player I wins is ~~4~~ $\frac{5}{9}$.

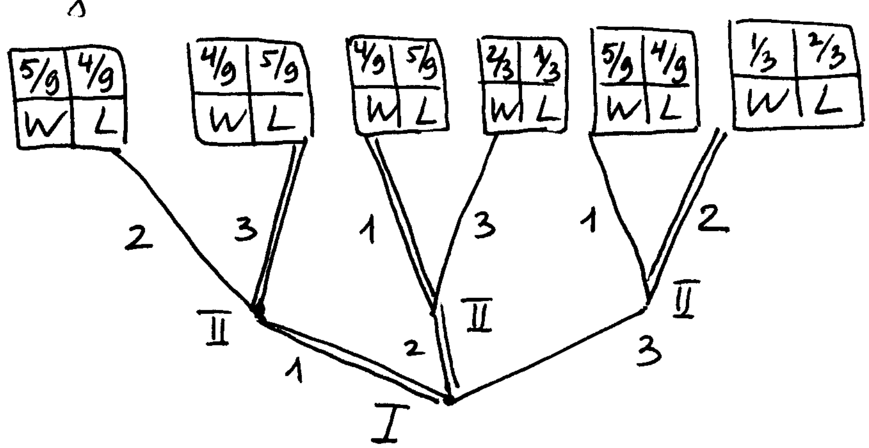
(b), (c)

7 max. score for (b)

5 max. score for (c)

(1, 3 1 2)
(2, 3 1 2) ⇐ Subgame-perfect equilibrium:

~~(wheel 1, wheel 3)~~ ~~(wheel 2, wheel 1)~~



Wheel 1 \ Wheel 3	2	4	6
1	L	L	L
3	W	L	L
9	W	W	W

$P(W) = 4/9$

Wheel 2 \ Wheel 3	2	4	6
0	L	L	L
7	W	W	W
8	W	W	W

$P(W) = 2/3$

Problem 4. An open-air concert organizer is worried about the possibility of rain, which is predicted to occur with the probability $p = 0.2$ (i.e., 20%) on the day of the concert. If the day is sunny, the profits of the concert will be \$100,000. However, if it rains, the profits will be only \$40,000. (In case of rain, the loss of the profits is \$60,000). The organizer considers buying an insurance. His utility for money function is $u(x) = \sqrt{x}$. The insurance company offers him an insurance for the full loss (\$60,000) in the case of rain. Such an insurance costs \$15,000. Will the organizer buy this insurance?

Solution.

We need to compare what happens if the organizer buys the insurance with what happens if he does not.

① If he does not buy an insurance, he gets the lottery

$$L = \begin{array}{|c|c|} \hline 100 & 40 \\ \hline 0.8 & 0.2 \\ \hline \end{array}$$

with $\mathcal{E}u(L) =$
$= 0.8 \cdot \sqrt{100} + 0.2\sqrt{40} = 8 + 0.2\sqrt{40}$

② With insurance, he always gets \$100,000 after the concert (either from the concert, if there is no rain, or from the concert + from insurance, if there is rain).

Thus, his gain in this case is
$X = 100 - 15 = 85$ (in thousands of dollars)

With insurance, he gets the lottery

$$M = \begin{array}{|c|c|} \hline 85 & 85 \\ \hline 0.2 & 0.8 \\ \hline \end{array}$$

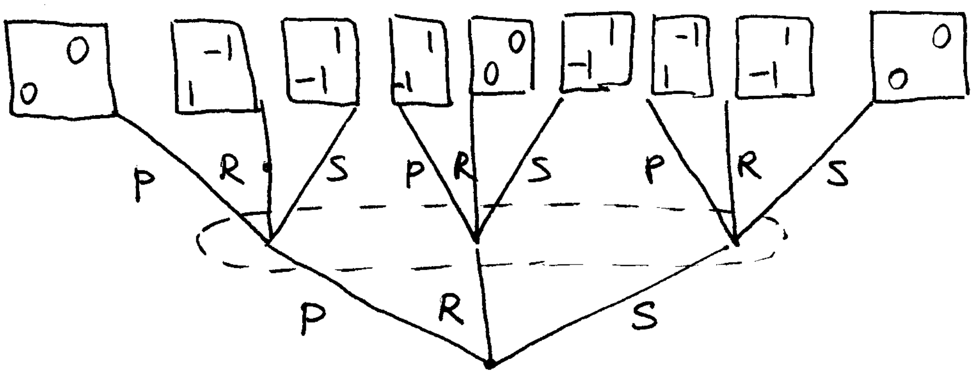
with $\mathcal{E}u(M) = 0.8\sqrt{85} + 0.2\sqrt{85} = \sqrt{85}$.

We need to compare $\sqrt{85}$ and $8 + 0.2\sqrt{40}$.
Square: 85 vs. $64 + 1.6 + 3.2\sqrt{40}$; 19.4 vs. $3.2\sqrt{40}$;
$\Longleftrightarrow$ 6.0625 vs. $\sqrt{40}$. Square again $\Rightarrow$ $6.0625 < \sqrt{40}$. $\Rightarrow$
$\mathcal{E}u(M) < \mathcal{E}u(L) \Rightarrow$ will not buy insurance

Problem 5. Two players play the children's game of *Paper-Rock-Scissors* as follows. On the count 1-2-3, they have to show a hand sign representing either paper, ~~or~~ rock, or scissors. Paper wins over Rock, but looses to Scissors. And Scissors loose to Rock. If they show the same sign, it's a draw. Assume that in the case of a draw, each gets a 0, in all other cases, the winner gets 1 and his opponent gets -1.

(a) Draw the game tree. (Note: this is a simultaneous-move game).
(b) Find the strategic form.
(c) List all dominated strategies.

(a) max. score 4



(b) max. score 4

I\II	P	R	S
P	0, 0	1, −1	−1, 1
R	−1, 1	0, 0	+1, −1
S	1, −1	−1, +1	0, 0

(c) There are no dominated strategies.

2 max. score