

Class 285I, Winter 2019: *Hilbert bimodules and subfactors of II_1 factors*

Instructor: Sorin Popa

Meetings: MWF 3-4pm in MS6118.

This is a topics course in Operator Algebras, which is conceived as a continuation of the 259A class in Fall 2018. We begin by reviewing some of the material covered in that class: the notion of Hilbert modules over type II_1 factors M and their dimension, the related notion of amplification of factors by positive real numbers, $M^t, t > 0$, and the fundamental group $\mathcal{F}(M)$.

Then we consider *Hilbert bimodules*, as a generalized notion of morphisms between II_1 factors. We show that bimodules over a II_1 factor M can be viewed as representations of M , with the completely positive maps playing the role of positive definite functions and a related GNS-type construction. We also consider “pointed versions” of these objects, associated with a II_1 factor and a subalgebra $B \subset M$, and an important related tool: *the basic construction*. We introduce Jones’ *index* of a bimodule, respectively of a subfactor, and prove his important theorem from 1982, showing that the index can only take values in the set $\{4 \cos^2 \theta \mid \theta \in (\0, \pi/3)\} \cup [4, \infty]$, with all values attained as index of subfactors of the hyperfinite II_1 factor R . Then we introduce the *standard invariant* and *graph* of a II_1 subfactor of finite index.

Finally, we’ll prove that any amenable II_1 factor M is isomorphic to R (Connes 1975 Theorem), with its subfactors with amenable graph completely classified by their standard invariants (Popa1994). This will allow us to give a complete list of all subfactors of R of index ≤ 4 .

All registered students in the class who will attend regularly will get A, but they are expected to make presentations in the 296I/290I Functional Analysis Seminars Mo 4-5:30/We 4-5:30, throughout the quarter.