

**Class 259B, Winter 2020:** *Classification results for  $\text{II}_1$  factors*

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**Meetings:** MWF 3-4pm in MS5118.

A central problem in von Neumann algebras is the classification of  $\text{II}_1$  factors  $L(\Gamma)$ ,  $L(\Gamma \curvearrowright X)$  arising from countable groups  $\Gamma$  and their measure preserving actions  $\Gamma \curvearrowright X$  on probability spaces  $(X, \mu)$ . These algebras tend to forget their building data, for instance, by a Theorem of Connes in [1], all  $\text{II}_1$  factors  $L(\Gamma)$ ,  $L(\Gamma \curvearrowright X)$  with  $\Gamma$  amenable are known to be isomorphic to the hyperfinite  $\text{II}_1$  factor  $R$ . But for non-amenable groups the situation can be quite rigid, for example the  $\text{II}_1$  factors associated with Bernoulli actions  $\Gamma \curvearrowright X = [0, 1]^\Gamma$  of property (T) groups  $\Gamma$  are isomorphic iff the groups  $\Gamma$  are isomorphic [3].

We'll first prove Connes Theorem in [1], using an approximation technique in [2]. We'll then develop some tools known as *deformation-rigidity theory*, that will allow us to prove the above rigidity result [3].

Some familiarity with Operator Algebra and  $\text{II}_1$  factors (255A&B, 259A) will be assumed (one can use <https://www.math.ucla.edu/~popa/Books/IIun-v14.pdf> to fill in the gaps). All registered students will get an A, but will have to make presentations in the Participating Seminar 290I Mondays 4-5:30 in MS5137.

- [1] A. Connes: *Classification of injective factors*, Ann. Math., **104** (1976), 73-115.
- [2] S. Popa: *A short proof that injectivity implies hyperfiniteness for finite von Neumann algebras*. J. Operator Theory, **16** (1986), 261-272.
- [3] S. Popa: *Strong Rigidity of  $\text{II}_1$  Factors Arising from Malleable Actions of  $w$ -Rigid Groups I, II*, Invent. Math., **165** (2006), 369-408, 409-453.