•**32.** (a) If $M_1 \subset M$ is a closed submanifold, $U \supset M_1$ is any neighborhood, and $f: M_1 \to \mathbb{R}$ is C^∞, then there is a C^∞ function $\tilde{f}: M \to \mathbb{R}$ with $\tilde{f} = f$ on M_1, and with support $\tilde{f} \subset U$.
(b) This is false if $M = \mathbb{R}$ and $M_1 = (0, 1)$.
(c) This is false if $f: M_1 \to \mathbb{R}$ is replaced by $f: M_1 \to N$ for a disconnected manifold N.

Remark: It is also false if $M = \mathbb{R}^2$, $M_1 = N = S^1$, and $f =$ identity; in fact, in this case, f has no continuous extension to a map from $\mathbb{R}^2$ to S^1, but the proof requires some topology. However, f can always be extended to a C^∞ function in a neighborhood of M_1 (extend locally, and use partitions of unity).

33. (a) The set of all non-singular $n \times n$ matrices with real entries is called $GL(n, \mathbb{R})$, the **general linear group**. It is a C^∞ manifold, since it is an open subset of $\mathbb{R}^{n^2}$. The **special linear group** $SL(n, \mathbb{R})$, or **unimodular group**, is the