On Pre-Catalan Catalan Numbers: Kotelnikow (1766)

In considering the first recordings of the well known Catalan sequence $\{1, 1, 2, 5, 14, 42, \ldots\}$ in the context of geometry, I and a colleague examined research conducted during the 18th and 19th centuries on a problem of polygon partitioning for which solutions are provided by the elements thereof — all decompositions of an n-gon into $n-2$ triangles by means of $n-3$ non-intersecting diagonals connecting its vertices are, for $n \geq 3$, enumerated by the $(n-1)$th Catalan number c_{n-2}, where

$$c_n = \frac{1}{n+1}\binom{2n}{n}, \quad n = 0, 1, 2, \ldots \tag{1}$$

We duly authored a *Mathematics Today* paper[1] which has already attracted considerable interest.

Omitted from our article was mention of a paper entitled "Demonstratio Seriei $\frac{4\cdot 6\cdot 10\cdot 14\cdot 18\cdot 22\cdots(4n-10)}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdots(n-1)}$ Exhibitae In Recensione VI. Tomi VII Commentariorum A.S.P.", published by S Kotelnikow in Volume 10 of the journal *Novi Commentarii Academiae Scientiarum Imperialis Petropolitanae* in 1766 (pp. 199–204). Strictly speaking, Kotelnikow could have been accorded reference since he was, along with one or two others, clearly aware of the problem and its solution well before Catalan. It seems only reasonable to emphasise that this contribution to the literature has been deemed to be negligible — lest its absence be seen as a major oversight on our part — but at the same time I now feel that for completeness an explanation in this Letter as to why is appropriate, and hopefully interesting because of the nature of the paper.

Prior to Kotelnikow's paper, only two formal articles had appeared on the triangular decomposition problem — one by Euler which contained the formula

$$\frac{2\cdot 6\cdot 10\cdot 14\cdot 18\cdot 22\cdots(4n-10)}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdots(n-1)} \tag{2}$$

for the c_{n-2} number of ways it can be done for an n-gon, and another one by von Segner which did not — both of which can be found in Volume 7 of the aforementioned journal from the year 1761.[2,3] Then, quite suddenly, Kotelnikow produces this offering in a subsequent volume (complete with typographical error in the mathematical expression of its title†). He states that his work arose from a desire to satisfy the general need for a pamphlet on the science of land surveying, from which he somehow dropped onto the polygon question as one of particular concern. We learn that Kotelnikow made only very limited progress in formulating (2) by simply constructing diagrams to show all possible cases of decomposition for low values of n (expressing uncertainty regarding the $c_5 = 42$ divisions for the heptagon), but he writes of having obtained it for himself by other means. Intimating to the reader that initially he was not sufficiently confident to put forward the work for publication, he continues rather brightly, a translation of which from Latin reads in essence (my thanks to colleague John Snell for this) as follows:

> "However, when I recently acquired Volume 7 of the Proceedings, as I wished to satisfy myself of the reliability of the series which I had found, I examined the paper of the distinguished Professor Segner in order to discover if there was anything in it which was to my benefit; when I found nothing of the sort, turning to the summary of the papers [of the volume] in order to look at the opinion expressed about it [i.e., von Segner's], I unexpectedly saw, to my greatest joy, the same series, expressed by a supreme geometrician, whose authority added the fullest possible weight to my proof, and left me utterly convinced of the reliability of the series. I believe too that he reached this series by means of the same calculations through which I had arrived at it. They are these:"

Upon examining his 'analysis', on the other hand, one sees that he has merely verified the result of that "supreme geometrician" Euler, and seemingly does little more than play around with the factored form of the solutions contained in (2) (that is, the Catalan numbers $c_1 = 1 = 2/2$, $c_2 = 2 = (2\cdot 6)/(2\cdot 3)$, $c_3 = 5 = (2\cdot 6\cdot 10)/(2\cdot 3\cdot 4)$, etc.) after having consulted and ignored the work of von Segner.[3] There is nothing which can be described as original or significant in the article and it would appear to have been condemned to suffer obscurity as a consequence, surprisingly receiving to my knowledge no recognition in any of the papers relevant to the topic which formed the basis of the study by myself and Paul Wilson — in addition to those already cited, these include works by Fuss (1795), Lamé (1838), Rodrigues (1838), Catalan (1838, 1839), Duhamel (1839), Grunert (1841), Tellkampf (1842), Binet (1839, 1843) and Liouville (1843) on mathematical aspects of this combinatorial problem and its natural generalisations. Collectively, they confirm the technically lightweight nature of Kotelnikow's paper, although it is only right and proper to formally document its existence since it is a rather anomalous one and of interest for this reason.

† This is repeated, with the addition of two more, in the contents listing of the article. Having said that, neither is the printing of the formula (2) in Euler's paper[2] error free; 18th century typesetting of mathematics was not an exact science.

References

1. Larcombe, P J and Wilson, P D C. (1998). On the trail of the Catalan sequence, *Math. Today*, **34**, 114–117.
2. Euler, L. (1761). Enumeratio modorum, quibus figurae planae rectilineae per diagonales diuiduntur in triangula, auctore I. A. de Segner, pag. 203, *Novi Comm. Acad. Sci. Imp. Pet.*, **7**, 13–15.
3. Von Segner, J A. (1761). Enumeratio modorum, quibus figurae planae rectilineae per diagonales diuiduntur in triangula, *Novi Comm. Acad. Sci. Imp. Pet.*, **7**, 203–210.

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Mathematics Teaching in China

On a recent visit to China I visited a mathematics lesson at JinLing Middle School in Nanjing, Jiansu Province. It was equivalent to Year 8 top set in an English school. There were 54 pupils in the class and they were studying quadratic equations.

A theoretical treatment considered "$b^2 - 4ac$" as positive, negative or zero and both factorising and completing the square were introduced as methods of solution. No mention was made of any geometrical interpretation.

Within minutes the teacher progressed from $x^2 + 4x - 12 = 0$ to $20x^2 - 39x + 18 = 0$ to $\frac{1}{3}x^2 - \frac{5}{3}x - 8$. The students tackled these confidently and without fuss (the boy sitting next to me was only fractionally behind me in getting the answer although he couldn't claim "jet lag" as an excuse!). I couldn't help contrasting this with the debilitating lack of practice and confidence evidenced in some of our A-Level students, never mind 13 year olds.

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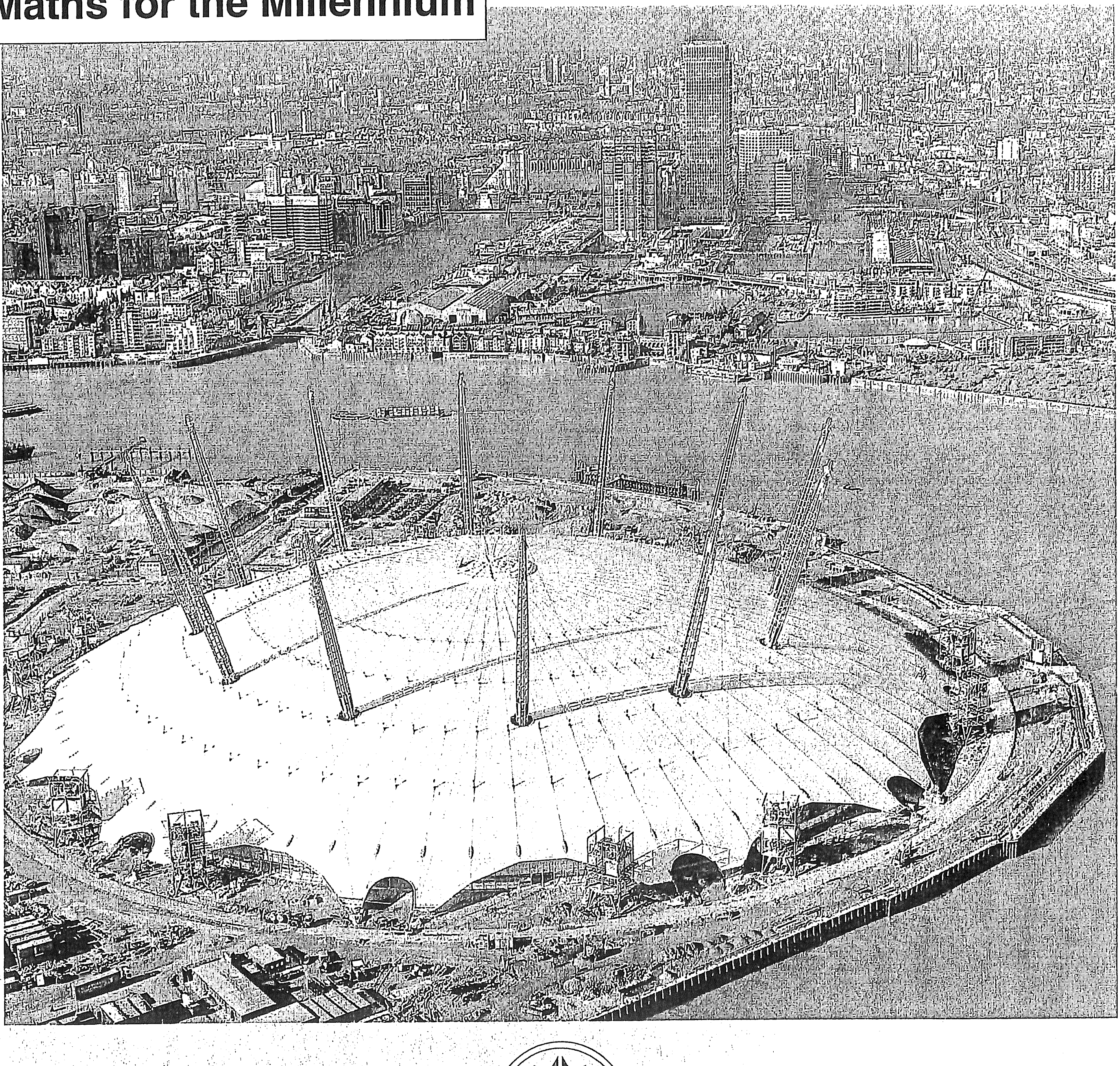

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