HOME ASSIGNMENT 4 (18.05, SPRING 2007)

Read: *Dekking et al.* Chapters 13, 14. See the table on p. 432.

Solve: Problems 13.6, 13.11, 13.12, 14.1, 14.2, 14.6, 14.10 (10 points each) and the following problems (each is worth 15 points).

Problem I. In condition of 14.1, let $Y = X_1 + \ldots + X_{144}$.

    a. Use the C.L.T. to approximate $P(Y \leq 240)$, $P(Y \geq 312)$, $P(Y \geq 348)$.

    b. Use Chebyshev inequality to bound the same probabilities. Compare the results.

Problem II. In condition of 14.8, let $Y = Y_{100}$.

    a. Use the C.L.T. to approximate $P(100 \leq Y \leq 110)$, $P(Y \geq 115)$, $P(80 \leq Y \leq 120)$.

    b. Use the C.L.T. to approximate $P(75 \leq Y \leq 125)$, $P(Y \leq 130)$, and $P(86 \leq Y \leq 118)$.

Problem III. Let X_i, $1 \leq i \leq 600$ be independent c.r.v. with $U(0,1)$ distribution. Let $Y_i = X_i(1 - X_i)$, $1 \leq i \leq 600$, and let $Z = Y_1 + \ldots + Y_{600}$.

    a. Compute $\mu = E[Y_i]$, $\sigma^2 = Var(Y_i)$, $E[Z]$ and $Var(Z)$.

    b. Use Chebyshev inequality to bound $P(Z \geq 103)$ and $P(Z \leq 98)$.

    c. Use the C.L.T. to approximate $P(100 \leq Z \leq 102.3)$, $P(Z \geq 103)$, $P(Z \leq 98)$, $P(92 \leq Z \leq 101.5)$.

Problem IV. Roll a fair die 200 times. Let X be the sum of the first 100 outcomes, and let Y be the sum of the second 100 outcomes.

    a. Use the C.L.T. to approximate $P(X \leq 300)$, $P(X \leq 370)$, $P(320 \leq Y \leq 425)$, $P(285 \leq Y \leq 325)$.

    b. Use the C.L.T. to approximate $P(X + Y \leq 620)$, $P(X + Y \leq 750)$, $P(660 \leq X + Y \leq 800)$, $P(X + Y \geq 900)$

    c. Use the C.L.T. to approximate $P(X - Y \leq 50)$, $P(X - Y \leq 100)$, $P(|X - Y| \leq 25)$, $P(|X - Y| \leq 10)$, $P(|X - Y| \leq 3)$.

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This Homework is due Wednesday March 21 at 4 pm. in 2-108 (UMO)