HOMEWORK 9 (18.314, FALL 2006)

1. Problem 8.2.6 (p. 284)

2. Three bus lines arrive to a bus stop every 10 min, every 15 min, and every 20 min, respectively. A person comes to a bus stop at a random time. What is the expected time until the first bus arrives?

3. Let G be a random graph on n vertices (out of 2^n possibilities). Denote by p_n the probability that G can be properly colored with 4 colors. Prove that $p_n \to 0$ as $n \to \infty$.

4. Let G be a random graph on n vertices (out of 2^n possibilities). Denote by q_n the probability that G contains a Hamiltonian cycle. Prove that $q_n \to 1$ as $n \to \infty$.

5. Denote by $B(n,k)$ the number of binary trees on n vertices with exactly k left edges. Prove by induction that

$$B(n,k) \;=\; \frac{1}{n}\binom{n}{k}\binom{n}{k+1}$$

6. Denote by $p(n)$ the number of integer partitions of n. Prove that $p(n) > n^{10}$ for n large enough.

7. Denote by $q(2n)$ the number of partitions of $2n$ into even parts, and by $r(2n)$ the number of partitions of $2n$ into parts so that each part is repeated even number of times. Prove that $q(2n) = r(2n) = p(n)$.

Example. Let $n = 4$. There are five partitions in each class:
$q(8) = 5$: (8), (6, 2), (4, 4), (4, 2, 2), (2, 2, 2, 2)
$r(8) = 5$: (4, 4), (3, 3, 1, 1), (2, 2, 2, 2), (2, 2, 1, 1, 1, 1), (1, 1, 1, 1, 1, 1, 1, 1)
$p(4) = 5$: (5), (4, 1), (3, 2), (3, 1, 1), (2, 1, 1, 1), (1, 1, 1, 1, 1)

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This Homework is due Friday December 8 at 2:05 pm.