MIDTERM 1 (MATH 61, SPRING 2017)

X 7 NT	
Vour Name	
Your Name:	

UCLA id:

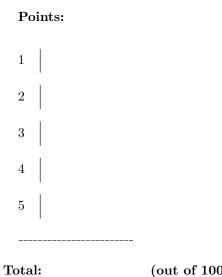
Math 61 Section: _____

Date: _____

The rules:

You MUST simplify completely and BOX all answers with an INK PEN. You are allowed to use only this paper and pen/pencil. No calculators. No books, no notebooks, no web access. You MUST write your name and UCLA id. Except for the last problem, you MUST write out your logical reasoning and/or proof in full. You have exactly 50 minutes.

Warning: those caught violating the rules get automatic 10% score deduction.





(out of 100)

Problem 1. (20 points)

Compute the probability that 4-subset A of $\{1, 2, \dots, 10\}$ satisfies:

- a) A has no odd numbers,
- b) A has at least one number ≤ 3 ,
- c) A contains 1 but not 7.
- d) the smallest number in A is divisible by 3

Problem 2. (20 points)

Let $X = \mathbb{N} = \{0, 1, 2, \ldots\}$ be the set of all non-negative integers. For each of the following functions $f : X \to X$ decide whether they are injective, surjective, bijective:

 $a) \quad f(x) = x + 1$

b)
$$f(x) = x^2 - 1$$

c)
$$f(x) = 2x$$

d) $f(x) = (x^2 + 2x)/(x+2)$

Problem 3. (15 points)

Let $a_n = 1111 \cdots 1$ (*n* ones). Suppose a_k is divisible by 97. Use induction to show that $a_{k \cdot n} = 0 \mod 97$, for all $n \ge 1$.

Problem 4. (15 points)

Find closed formulas for the following sequences :

- a) $4, 4, 6, 8, 12, 18, 28, 44, 70, 112, \ldots$
- b) $a_1 = 1, \ a_{n+1} = a_n \cdot \binom{n+1}{2}$
- c) $a_1 = 1, a_2 = 1, a_{n+1} = a_{n-1} a_n$ for $n \ge 2$.

Note: you can express a_n in terms of Fibonacci numbers F_n .

Problem 5. (30 points, 2 points each) TRUE or FALSE?

Circle correct answers with ink. No explanation is required or will be considered.

- **T F** (1) The number of functions from $\{A, B, C, D\}$ to $\{1, 2, 3\}$ is equal to 4^3 .
- **T F** (2) The sequence 1, 3/2, 5/3, 7/6, 9/8, ... is increasing.
- **T F** (3) The sequence $-1, -2, -3, -4, \dots$ is non-increasing.
- $\mathbf{T} \in \mathbf{F}$ (4) There are 4 anagrams of the word MAMA.
- $\mathbf{T} = \mathbf{F}$ (5) There are infinitely many Fibonacci numbers which are divisible by 3.
- **T F** (6) The number of permutations of $\{1, 2, 3, 4, 5\}$ is smaller than 123.
- **T F** (7) The number of 3-permutations of $\{1, 2, 3, 4, 5, 6\}$ is equal to $\binom{6}{3}$.
- **T F** (8) The number of 3-subsets of $\{1, 2, 3, 4\}$ is equal to 4.
- **T F** (9) The number of permutations of $\{1, 2, ..., n\}$ which have *n* preceding n 1 (not necessarily immediately) is equal to n!/2
- $\mathbf{T} \quad \mathbf{F} \quad (10) \quad \text{For every } A, B \subset \{1, 2, \dots, 12\} \text{ we have } |A \cap B| < |A \cup B|.$
 - **F** (11) For all $n \ge 1$, we have $\binom{2n}{0} + \binom{2n}{2} + \binom{2n}{4} + \ldots + \binom{2n}{2n} = 2^{2n-1}.$
- **T F** (12) The number of grid walks from (0,0) to (10,10) going through (3,7) is equal to $\binom{10}{3}^2$.
- **T F** (13) The number of grid walks from (0,0) to (10,10) avoiding (10,0) and (0,10) is equal to $\frac{1}{2} \binom{20}{10}$.
- **T F** (14) The number of anagrams of MISSISSIPPI which begin with M is greater than the number of anagrams which begin with S.
- $\mathbf{T} = \mathbf{F}$ (15) The following parabolas are drawn in the plane:

$$y = x^2 - n^2 x - n^3$$
, $n = 1, \dots, 12$.

Then the regions of the plane separated by these parabolas can be colored with two colors in such a way that no two adjacent regions have the same color.

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