MIDTERM 2 (MATH 61, SPRING 2015)

Your Name: ____

UCLA id: ____

Math 61 Section: ____________

Date: ________________________

The rules:
You MUST simplify completely and BOX all answers with an **INK PEN**.
You are allowed to use only this paper and pen/pencil. No calculators.
No books, no notebooks, no web access. You MUST write your name and UCLA id.
Except for the last problem, you MUST write out your logical reasoning and/or proof in full. You have exactly 50 minutes.

Warning: those caught writing after time get automatic 10% score deduction.

Points:

1	
2	
3	
4	
5	

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Total: **(out of 100)**

Problem 1. (20 points)

Compute the number of (shortest) grid walks from $(0, 0)$ to $(9, 9)$ which:

a) do not go through any of the other diagonal points $(1, 1)$, $(2, 2)$, $\ldots$, $(8, 8)$

b) stay on or above $y = x - 1$ diagonal

c) stay on or above $y = x$ diagonal AND do not go through $(6, 6)$

d) stay on or above $y = x$ diagonal AND on or below $y = x + 1$ diagonal.

Note: You can use the formula for Catalan numbers or write the answer in terms of C_n's.

$$C_n = \frac{1}{n+1}\binom{2n}{n}.$$

Problem 2. (20 points)

Compute the number of subgraphs of G isomorphic to H, where

a) $G = K_{7,9}$, $H = C_4$

b) $G = K_{7,9}$, $H = P_4$

c) $G = C_9$, $H = P_4$

d) $G = K_9$, $H = K_{2,3}$

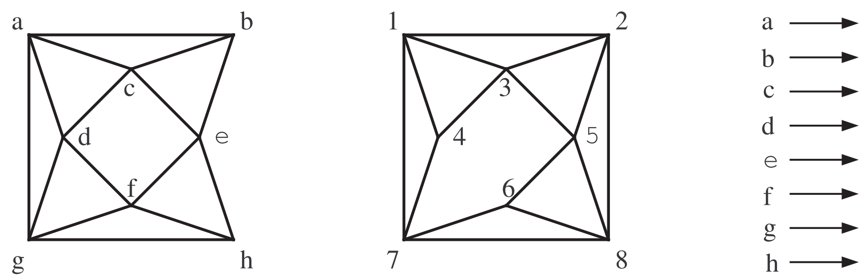
Problem 3. (15 points)

Let $a_1 = 2$, $a_2 = 7$, $a_{n+1} = a_n + 2a_{n-1}$. Solve this LHRR and find a closed formula for a_n.

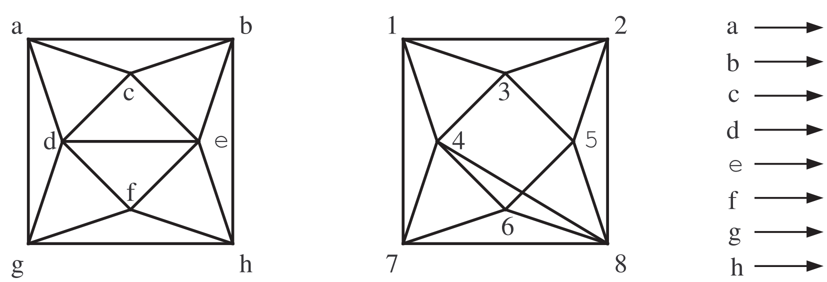
Problem 4. (15 points)

Decide whether the following pairs of graphs on 8 vertices are isomorphic or non-isomorphic.

a)



b)



Important: In case of isomorphism, you must write a bijection in the figure above (in ink). No need for further arguments. In case non-isomorphism, you must say so and present an argument why two graphs are not isomorphic.

Problem 5. (30 points, 2 points each) **TRUE or FALSE?**

Circle correct answers with ink. No explanation is required or will be considered.

T **F** (1) Isomorphic graphs have the same number of edges.

T **F** (2) Isomorphic graphs have the same number of connected components.

T **F** (3) Isomorphic graphs have the same number of 4-cycles.

T **F** (4) $F_n \le C_n$ for all integer n.

T **F** (5) Sequence $(3,3,3,3,3)$ is a valid score of a simple graph.

T **F** (6) Sequence $(4,4,4,4,2)$ is a valid score of a simple graph.

T **F** (7) Sequence $(4,4,4,2,2)$ is a valid score of a simple graph.

T **F** (8) Sequence $(4,4,2,2,2)$ is a valid score of a simple graph.

T **F** (9) Sequence $(2,2,2,0,0)$ is a valid score of a simple graph.

T **F** (10) Graph C_8 is a subgraph of $K_{7,7}$.

T **F** (11) Graph C_8 is a subgraph of $K_{9,3}$.

T **F** (12) Graph P_8 is a subgraph of $K_{9,3}$.

T **F** (13) Graph K_4 is a subgraph of $K_{7,7}$.

T **F** (14) Graph K_9 has 72 edges.

T **F** (15) Catalan numbers modulo 2 are periodic with period 6.