Dear Prof. Schacher,

Hi - I was thinking of you a few days ago, because it seems that there is a small improvement that can be made to your legendary first year grad. algebra notes!

This concerns the last thing you taught us in the group theory section - namely that S_n (symmetric group on n letters) is complete i.e. $\mathrm{Inn}(S_n) = Aut(S_n)$ (except suprisingly when $n = 6$, in which case there is an index 2 contaiment).

The way you prove this is by establishing two propositions. The first one claims that any auto of S_n that preserves the set of transpositions must be inner. You give a proof by induction here, which seems a little messy. (The second proposition seems fine!).

Instead, perhaps once could construct the 'conjugating element' directly. That is if f is the auto., then we will construct some g directly and show $f(x) = gxg^{-1}$.

The idea is simple. For instance, to see where g sends 1, pick any two transpositions containing 1, say (1 2) and (1 3). Say $f(1\ 2) = (i\ j)$ and $f(1\ 3) = (k\ l)$. Since (1 2)(1 3) is a three cycle $(i\ j)(k\ l)$ must have order three and so one of i or j must be equal to k or l. WLOG say $i = k$. Then let g map 1 to $i = k$.

Thus we know where g sends 1. We may repeat this procedure to construct g completely. (There are some small easy checks that must be made to ensure that everything is independent of all choices!)

Now one sees that on all transpositions x, $f(x) = gxg^{-1}$ and since the transpositions generate S_n, we are done.

All this came up because of the following interesting aside. The level 2 Sigel three fold (i.e. Sigel upper half three space modulo the level two congruence subgroup sitting inside $Sp_4(Z)$) has a compactification consisting of 30 boundary components: 15 curves and 15 cusps (points!). Somehow the action of the congruence subgroup sets up a bijection between the cusps and the curves, thereby somehow leading to a construction of a non inner auto. of S_6. Note here that 15 is exactly the number that arises from the coincidental fact that in S_6 there are 15 transpositions, and 15 elements that are products of three transpositions...

Do you know any other construction of a non-inner auto. of S_6?

By the way - I am now a postdoc at TIFR in Bombay, which is a great place...!

Sincerely and best wishes,

Eki Ghate.