HOMEWORK #5: ADAMS SPECTRAL SEQUENCE I

(1) Show that $\mathcal{A}(1)$ is an 8-dimensional algebra over $\mathbb{F}_2$ generated by 1, Sq^1, Sq^2, Sq^3, Sq^2Sq^1, Sq^2Sq^3, Sq^3Sq^1, and Sq^3Sq^3.

(2) Let HZ be the $\mathcal{A}(1)$-module $\mathcal{A}(1) \otimes_{\mathcal{A}(0)} \mathbb{F}_2$, where $\mathcal{A}(0)$ is the subalgebra generated by Sq^1. Compute $Ext^{s,t}_{\mathcal{A}(1)}(HZ, \mathbb{F}_2)$ for all s and t. Do this two ways: first by actually computing a minimal resolution and second by using a change-of-rings.

(3) Show that if P_* is a minimal resolution of M, then $Ext^s_{\mathcal{A}(1)}(M, \mathbb{F}_2)$ is just $Hom_{\mathcal{A}(1)}(P_s, \mathbb{F}_2)$.

(4) Let $C(\eta)$ be the $\mathcal{A}(1)$-module generated by a class a subject to the relations $Sq^1(a) = 0 = Sq^3(a)$. Show that as an $\mathbb{F}_2$-vector space, $C(\eta)$ is two dimensional, and compute $Ext_{\mathcal{A}(1)}(C(\eta), \mathbb{F}_2)$.

(5) We saw that the cohomology of the integral Eilenberg-MacLane spaces was characterized by the additional relation: $Sq^1(\iota_n) = 0$. This infinitely deloops to show us that $H^*(H\mathbb{Z}) = \mathcal{A} \otimes_{\mathcal{A}(0)} \mathbb{F}_2$. Use this and the Adams spectral sequence to show that the class corresponding to Sq^1 in $Ext^{1,1}$ detects multiplication by 2.

(6) Building on the previous problem, run the Adams spectral sequence for $H\mathbb{Z}/2^k$ as $k \geq 1$. You should see that the E_2 terms are all isomorphic for $k > 1$, and the only difference is in the differentials. You may use the fact that a short exact sequence of modules induces a long exact sequence in Ext.