Math 113
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Let me know of any mistakes

Problem 3.3.3: b is 3-colorable. the chromatic number of a, c and d is 4, the chromatic number of b is 3
Problem 3.3.6: Yes, have B and C, and D and E at the same times.
Problem 3.3.8: four
Problem 3.3.13: Both are yes. This was a midterm problem, since the number of vertices is 17 and $\chi = 4$ this implies the independence number $> 17/4$. since the independence number is a whole number this implies the independence number is at least 5. So there is an independent set with at least 5 vertices which implies (by taking subsets of this independent set) that there are independent sets with 5 and 4 vertices.
Problem 3.3.59: since a graph can be colored with $\chi(G)$ colors, pick some $\chi(G)$ coloring and let all the vertices with color 1 be given the first k colors in the k-tuple coloring, let all the vertices with color 2 be given the second k colors in the k-tuple coloring, and so on. Thus we have a k-tuple coloring with $k * \chi(G)$ colors, So the minimum number of colors needed for a k-tuple coloring is less than $k * \chi(G)$.
Problem 3.4.3: $x(x-1)^4$
Problem 3.4.6: $P(I_4, x) - 5P(I_3, x) + 8P(I_2, x) - 4P(I_1, x)$
Problem 3.4.14: Clearly no graph can be zero colored so $P(G, 0) = 0$, that is the constant term is zero. Only one family of graphs can be 1-colored, the graph with no edges. In this case $P(G, x) = x^n$ where n is the number of edges, otherwise $P(G, 1) = 0$ that is the sum of the coefficients is zero.
Problem 3.4.15: We prove it by induction on the number of edges e. Clearly if there are 0 edges then $P(G, x) = x^n$ where n is the number of vertices. Clearly the degree is n and leading coefficient is 1. This is the basis of our induction. Now assume that for graphs with less than N edges $P(G, x)$ has degree n and leading coefficient 1. Let G be a graph with N edges, pick an edge α and use the fundamental reduction theorem on the edge α. Then $P(G, x) = P(G'_\alpha, x) - P(G''_\alpha, x)$ and both G'_α and G''_α have less than N edges and G''_α has $n-1$ vertices while G'_α has n vertices. Thus $P(G'_\alpha, x)$ is a polynomial of degree n with leading term x^n and $P(G''_\alpha, x)$ is a polynomial of degree $n-1$ with leading term x^{n-1}.

So $P(G, x) = P(G'_\alpha, x) - P(G''_\alpha, x)$ is a polynomial of degree n with leading coefficient 1. Thus completing our proof by induction.
Problem 3.4.21: $P(W_n, x) = xP(Z_n, x-1)$
Problem 13.3.4: a and b are augmented chains
Problem 13.3.21: 600 barrels/hour
Problem 11.1.12.a: assuming 7 digit phone numbers there are 10^7
b: $42 * 12 * 10^7$
Problem 11.2.10: If a graph has a bridge, then the vertices then bridge connects are cut vertices. Thus having a bridge implies having cut vertices which also means having no cut vertices implies having no bridges. Since our graph is

connected and has no cut vertices this implies it is connected and has no bridges thus it has a strongly connected orientation.
Problem 11.2.14.a: 2^{5n+2} b: 2^{126} c: 2^{48}