Math 113
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Let me know of any mistakes

Problem 2.1.2: there are enough codewords for $(26)^3 * 10^3 > 5000$

Problem 2.1.5: need $2^1 + 2^2 + \ldots 2^n \geq 36$ so $n = 5$

Problem 2.1.8: $10^6 - 9^6$ numbers x s.t. $0 \leq x \leq 999999$ that have no twos.

Problem 2.1.11: $2^{(2^{(n-1)}}$ switching functions

Problem 2.2.1: $8 + 16 + 32$ bitstrings

Problem 2.2.2: $8 * 7 + 8 * 12 + 7 * 12$ commitees

Problem 2.2.6: $2 + 3$ rolls

Problem 2.2.7: 21^{30} possibilities

Problem 2.3.2: 4!

Problem 2.3.5: 4!/2

Problem 2.3.6.a: 4!
b: $(n - 2)!$

Problem 2.5.2.a: $6 * 6 * 6$
b: $6 * 5 * 4$
c: $1 * 6 * 6$
d:$1 * 5 * 4$

Problem 2.5.3.a: 8^4
b: $8 * 7 * 6 * 5$
c: $1 * 8 * 8 * 8$
d: $1 * 6 * 5 * 1$

Problem 2.6.2: there are $2^{35} - 1$ non-empty subsets so we need to find n so that $2^{35} - 1 < 10^n$ so $n = 12$.

Problem 2.6.7.a: 2^8
b: $2^{(2^n)}$

Problem 2.7.4: n

Problem 2.7.15: $5 * 4 * 3 * 2$

Problem 2.7.21: sum of the rows of the triangle. general answer is 2^n

Problem 2.14.1.a:$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$
c: $(2u)^4 + 4(2u)^3(3v) + 6(2u)^2(3v)^2 + 4(2u)(3v)^3 + (3v)^4$

Problem 2.14.3: $C(12,5)C(4,4)+C(12,6)C(4,3)+C(12,7)C(4,2)+C(12,8)C(4,1)+C(12,9)C(4,0) = 10648$

Problem 2.8.3.a: 3/8
b: 4/8
c: 6/8

Problem 2.8.5: $1 - 1/16$

Problem 2.8.9: 5/16

Problem: 2.8.10: $1 - 3/8$

Problem: 2.8.13: $(1 - 1/8)(1 - 1/4)$

Problem 2.9.8.a: $C^R(3, 82) = C(84, 82) = 3486$
b: $C^R(2, 82) = C(83, 82) = 83$

Problem 2.9.10: $\Sigma_{i=0}^{n} C(i + m - 1, i)$
Problem 2.10.8.a: 18
b: 16

Problem 2.10.18: 6 marbles, 4 cells no empty cells. Choose distinguished/undistinguished and use table.

Problem 2.10.24.a: 16
b: 4
c: precisely the same as dist. cells, undist marbles, cells must be filled.

Problem 5.1.2.f: $\Sigma_{k=0}^{\infty}(k + 1)(k + 2)/2x^k$
l: $\Sigma_{k=1}^{\infty}(-1)^{k+1}x^{2k}/k$

Problem 5.1.3.f: $\frac{4}{1-x} - 4 - 4x$
h: $3e^x$
n: $\frac{1}{1-x^2}$

Problem 5.1.4.m: $a_k = (-2)^k/k!$
Problem 5.2.1.f: $a_0 = 4, a_k = 4 + 7^{k-1}$ for $k > 0$
g: $a_0 = 6, a_1 = 6, a_k = 1$ $k \geq 2$ even, $a_k = 0$ for $k \geq 2$ odd.

h: the sinh(x) $a_k = 0$ for k even , $a_k = 1/k!$ for k odd.

Problem 5.2.3.c: $(1, 1, 3, 5, 7, 9, \ldots)$

Problem 5.2.6: $1 * 12 - 5 * 10$

Problem 5.3.1.a: $(1 + x + x^2 + x^3)^2(1 + x + x^2)^2$, x^5 coefficient
b: $(1 + x^4)(1 + x)^3$, x^5 coefficient
c: $(x^3 + x^4 + \ldots + x^7)(x^3 + x^4 + \ldots + x^6)(x^2 + x^3 + x^4)$, x^{11} coefficient
e: $(x^4 + \ldots + x^7)(1 + x + \ldots x^6)^2$, x^{12} coefficient
f: $(1 - x)^{-5}$, x^{20} coefficient
g: $(1 - x)^{-4}$, x^{40} coefficient
h: $(1 - x)^{-100}$, x^{40} coefficient
i: $(1 - x)^{-8}$, x^{12} coefficient
j: $(\frac{x}{1-x})^8$, x^{12} coefficient
k: $(x + x^2 + \ldots + x^6)^{14}$, x^{30} coefficient
l: $(1 - x)^{-60}$, x^{10} coefficient
m: $(1+x+x^2+\ldots)(1+x^5+x^{10}+\ldots)(1+x^{10}+x^{20}+\ldots)(1+x^{25}+x^{50}+\ldots)$, x^{100} coefficient
n: $(1 + x + x^2 + \ldots + x^7)^3$, x^{14} coefficient
o: $(x+x^3+x^5+\ldots)(x^2+x^3+x^4+x^5)(x^2+x^3+x^5+x^7+x^11+\ldots)$, x^{20} coefficient

Problem 5.3.4: $\Pi_{i=1}^{p}(1 + x^{n_i}$

Problem 5.3.14.a: 5
d: $\Pi_{k=1}^{\infty} \frac{1}{1-x^{2k-1}}$

Problem 5.3.17.a: 56
Problem 5.3.17.d: 1001

Problem 5.4.2: find the coefficient of x^7 in the expansion
a: $x^3(1-x)^{-4}$, this is the same as finding the coefficient of x^4 in $(1-x)^{-4}$ which is $\binom{-4}{4} = (-4)(-5)(-6)(-7)/4! = 35 = \binom{4+4-1}{4}$.
c: $x^2(1 + x^2)^3(1 - x)^{-3} = x^2(1 + 3x^2 + 3x^4 + x^6)(1 + \binom{3+1-1}{1}x + \binom{3+2-1}{2}x^2 + \binom{3+3-1}{3}x^3 + \binom{3+4-1}{4}x^4 + \binom{3+5-1}{5}x^5 + \ldots) = (x^2 + 3x^4 + 3x^6 + x^8)(1 + 3x + 6x^2 + 10x^3 + 15x^4 + 21x^5 + \ldots)$. So the coeeficient on x^7 is $1 * 21 + 3 * 10 + 3 * 3 = 60$.

Problem 5.4.6: How many ways is there to choose 11 personal computers if 6 models are available?
We have 6 types of things, the computers, and we can choose any number of computers. So our generating function would be $(1+x+x^2+x^3+x^4+\ldots)^6 = (1-x)^{-6}$ and we want to find the coefficient of x^{11}. Using the Binomial series this is just $(-1)^{11}\binom{-6}{11} = \binom{6+11-1}{11} = 4368$. This the number of way of picking the computers if we consider all the computers made a manufacturer indistinguishable and the order in which we pick doesn't matter. If we had used exponential generat-

ing functions instead it would have given us the answer if computers from the same manufacturer were distinguishable and types are distinguished.

Problem 5.4.7: How many ways are there to choose 50 shares of stock if 4 companies each have 100 shares available?
We have 4 types each offering frome 0 to 100 shares so the ordinary generating function is $(1 + x + x^2 + \ldots + x^{100})^4$ and we want the x^{50} coefficient. Since 101¿50 we might as well let our generating function be $(1 - x)^{-4}$ as the answer will be the same and it will be easier to compute. Thus the coefficient of x^{50} will be $\binom{4+50-1}{50} = 23426$. This is if we're counting with stock within companies indistinguishable, and order not mattering.

Problem 5.5.1.a: $5e^x$
b: e^{3x}
c: $e^x - x - x^2/2!$
d: $e^x - 1 - x$
e: $cosh(x) = \frac{e^x + e^- x}{2}$
f: $e^x + cosh(x)$

Problem 5.5.6.a: $(e^x)^2(1 + x/1!)^2$, $x^3/3!$ coeeficient
b: $(e^x - 1)^5$, $x^6/6!$ coeeficient
c: $(1+x+x^2/2!+x^3/3!+x^4/4!)^2(1+x+x^2/2!+x^3/3!)(e^x)^7$, $x^{11}/11!$ coeeficient
d: $(1 + x + x^2 + \ldots x^{2n})^3$, x^{3n} coefficient
e: $(1 + x^2/2! + x^4/4! + \ldots)^2(x/1! + x^3/3! + x^5/5! + \ldots)$, $x^n/n!$ coeeficient. Note since we assume n is even this implies there are 0 ways to do it. f: $(x + x^2 + x^3 + x^4 + x^5 + x^6)^{50}$, x^{100} coeeficient
g: $(1 - x)^{-3}$, x^{10} coefficient if bonds are indistinguished, $(e^x)^3$, $x^{10}/10!$ if bonds are distinguished
h: $(1 + x + x^2/2! + \ldots x^5/5!)^2(1 + x + x^2/2! + \ldots x^{15}/15!)$, $x^{20}/20!$ coeeficient
i: $(x + x^2)^3(x^2 + x^3)^5$, x^12 coeeficient
j: $(x^{20} + x^{40} + x^{60} + x^{80} + x^{100})^4$, x^{200} coeeficient
k: $(1 + x + x^2/x! + \ldots + x^5/5!)(1 + x + x^2/x! + \ldots + x^4/4!)^2(1 + x)$, $x^{12}/12!$ coeeficient
l: same with answer sum of $x^{12}/12! + x^{13}/13! + x^{14}/14!$ coefficients. Note we stop at 14 since there are only 14 flags.

Problem 5.5.19.a: $e^x(1 + x)^6$, find a_3

$e^x(1 + x)^6 = (1 + x + x^2/2! + x^3/3! + \ldots)(1 + 6x + 15x^2 + 20x^3 + \ldots)$ so the x^3 coefficient is $(20 + 15 + 6/2 + 1/6)x^3 = (120 + 90 + 18 + 1)x^3/3!$, so the coefficient is 229.

b: $\frac{e^{3x}}{1-x} = (1+3x+9x^2/2!+27x^3/3!+\ldots)(1+x+x^2+x^3+\ldots)$so x^3 coefficient is $(1 + 3 + 9/2 + 27/6)x^3 = (6 + 18 + 27 + 27)x^3/3!$ so the coefficient is 78.

c: $\frac{x^2}{(1-x)^2} = x^2(1+2x+3x^2+4x^3+\ldots) = (2x^2/2!+12x^3/3!+72x^4/4!+\ldots)$ so the coefficient is 12.