HOMEWORK 5

1. Prove that if a nilpotent matrix A is diagonalizable, then $A = 0$.

2. Prove that if a matrix A is diagonalizable, then so is A^2.

3. Suppose that $f(T) = 0 = g(T)$ for a linear operator T. Prove that the polynomials f and g are not relatively prime.

4. Let A be an $n \times n$ matrix with n distinct eigenvalues. Prove that A is diagonalizable.

5. Find the Jordan canonical form and the Jordan canonical basis for the linear operator on $P_2(\mathbb{R})$ defined by $T(f) = 2f - f'$.

6. Find the Jordan canonical form and the Jordan canonical basis for the linear operator on $M_{2\times 2}(F)$ defined by $T(A) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot A$.

7. Find the Jordan canonical form and the Jordan basis for the linear operator on $M_{2\times 2}(F)$ defined by $T(A) = 2A + A^t$.

8. Find the Jordan canonical form of the matrix $\begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & -1 & 3 \end{pmatrix}$

9. Let J be the Jordan canonical form of a linear operator T. Find the Jordan canonical form of the operator aT, where a is a nonzero scalar.

10. Let W be a subspace of a vector space V. Let $T : V \to V/W$ be the linear operator defined by $T(v) = \bar{v}$. Consider the dual operator $T^* : (V/W)^* \to V^*$. Prove that $N(T^*) = \{0\}$ and $R(T^*)$ coincides with the null space of the restriction operator $V^* \to W^*$.