HOMEWORK 2

1. Determine whether the subset $\{(1,2),(2,1)\}$ is basis for $\mathbb{R}^2$.

2. The vectors $u_1 = (2,-3,1)$, $u_2 = (1,4,-2)$, $u_3 = (-8,12,-4)$, $u_4 = (1,37,-17)$, $u_5 = (-3,-5,8)$ generate $\mathbb{R}^3$. Find a subset of the set $\{u_1,u_2,u_3,u_4,u_5\}$ that is a basis for $\mathbb{R}^3$.

3. Let $\{u,v\}$ be a basis for V. Show that $\{2u+3v, u+2v\}$ is also basis.

4. Let W_1 and W_2 be two subspaces of a vector space V. Show that $\dim(W_1 \cap W_2) = \dim(W_1)$ if and only if $W_1 \subset W_2$.

5. Find the dimension of the spaces $\mathit{Sym}_{n\times n}(\mathbb{R})$ and $\mathit{Skew}_{n\times n}(\mathbb{R})$ of symmetric and skew-symmetric $n \times n$ matrices respectively.

6. Prove that the subset of F^n consisting of all vectors $(a_1,a_2,\ldots,a_n)$ such that $a_1+a_2+\ldots+a_n = 0$ is a subspace of F^n and find its dimension.

7. Prove that the subset of $P_n(F)$ consisting of all polynomials f such that $f(1) = 0$ is a subspace of $P_n(F)$ and find its dimension.

8. Let V be a finite dimensional vector space and $S \subset V$ a subset (possibly infinite) with $span(S) = V$. Prove that some subset of S is a basis for V.

9. Let W_1 and W_2 be finite dimensional subspaces of a vector space V. Prove that the subspaces $W_1 \cap W_2$ and $W_1 + W_2$ are also finite dimensional and

$$\dim(W_1 \cap W_2) + \dim(W_1 + W_2) = \dim(W_1) + \dim(W_2).$$

10. (*) Let V be a vector space of dimension n and let $V_1, V_2, \ldots, V_k \subset V$ be subspaces of V. Assume that

$$\sum_{i=1}^{k} \dim(V_i) > n(k-1).$$

Prove that $\bigcap_{i=1}^{k} V_i \neq \{0\}$.