HOMEWORK 9

1. Let T be an invertible linear operator on a finite dimensional inner product space. Prove that T^* is also invertible and $(T^{-1})^* = (T^*)^{-1}$.

2. Let T be a linear operator on an inner product space. Prove that $N(T^* \circ T) = N(T)$.

3. Let T be a linear operator on a finite dimensional inner product space. Prove that $rank(T^*) = rank(T)$.

4. Let T be a linear operator on a finite dimensional inner product space. Prove that $R(T^*)^\perp = N(T)$ and $R(T^*) = N(T)^\perp$.

5. Let T be a linear operator on an inner product space. Let $W \subset V$ be a subspace such that $T(W) \subset W$. Prove that $T^*(W^\perp) \subset W^\perp$.

6. Let $T : V \to W$ be a linear map of inner product spaces such that $\langle T(v), T(v') \rangle = \langle v, v' \rangle$ for all $v, v' \in V$. Prove that T is injective.

7. Inner product spaces V and W are called *isomorphic* if there is an isomorphism $T : V \to W$ such that $\langle T(v), T(v') \rangle = \langle v, v' \rangle$ for all $v, v' \in V$. Prove that every n-dimensional inner product space over F is isomorphic to F^n with the standard inner product.

8. Let T be a normal linear operator on a finite dimensional inner product space. Prove that $N(T^*) = N(T)$ and $R(T^*) = R(T)$.

9. Let T be a normal linear operator on a finite dimensional real inner product space V. Suppose that the characteristic polynomial of T splits. Prove that there is an orthonormal basis for V consisting of eigenvectors of T.

10(*). Let T be a normal linear operator on a finite dimensional complex inner product space V. Let $W \subset V$ be a subspace such that $T(W) \subset W$. Prove that $T^*(W) \subset W$.