HOMEWORK 2

1. Determine the degree of the extension $\mathbb{Q}(\sqrt{2}, \sqrt[3]{3})$ over $\mathbb{Q}$.

2. Let $F = \mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$. Prove that $\sqrt[3]{2} \notin F$.
(Hint: Show that the degree $[F(\sqrt[3]{2}) : \mathbb{Q}]$ is divisible by 3.)

3. Determine the degree of the extension $\mathbb{Q}\big(\sqrt{3 + 2\sqrt{2}}\,\big)$ over $\mathbb{Q}$.

4. Determine the splitting field of $X^4 - 2$ and its degree over $\mathbb{Q}$.

5. Determine the splitting field of $X^6 - 4$ and its degree over $\mathbb{Q}$.

6. Prove that an algebraically closed field is infinite.

7. Construct a field of 9 elements and give its addition and multiplication tables. Find a generator of the multiplicative group. How many generators are there?

8. Factor $X^8 - X$ into a product of irreducibles in $\mathbb{Z}[X]$ and $\mathbb{F}_2[X]$.

9. Let f be an irreducible polynomial of degree n over a finite field $\mathbb{F}_q$. Prove that f divides $X^{q^n} - X$.

10. Determine all q such that -1 is a square in $\mathbb{F}_q$.