HOMEWORK 8

1. Show that the group $\mathbb{Q}/\mathbb{Z}$ cannot be generated by a finite set of elements.

2. A *commutator* of a group G is an element of the form $xyx^{-1}y^{-1}$ where $x, y \in G$. Let G' be the subgroup of G generated by all commutators. We call G' the *commutator subgroup* of G. Prove that G' is normal in G.

3. Let G' be the commutator subgroup of a group G. Prove that the group G/G' is abelian.

4. Let N be a normal subgroup of a group G such that the factor group G/N is abelian. Prove that the commutator subgroup of G is contained in N.

5. Determine all Sylow 2-subgroups of the alternating group A_5.

6. Find the number of all Sylow p-subgroups of the symmetric group S_p (p is prime).

7. Prove that all groups of order $2p^n$ (p is prime, $n \geq 1$) are not simple.

8. Let $H \subset G$ be a subgroup. Prove that if H is contained in the center of G and the factor group G/H is cyclic, then G is abelian.

9. Prove that any group of order p^2 is abelian (p is prime). (Hint: Use Problem 8.)

10. Let G be a group of order p^3 (p is prime). Prove that the center of G contains the commutator subgroup of G.