HOMEWORK 6

1. Let p be a prime integer. Find the number of elements of order p in S_p.

2. Let p be a prime integer. Find the number of subgroups of order p in S_p.

3. Determine all n such that the n-cycle $(1, 2, \ldots, n)$ is even.

4. Let $H_1 \subset G_1$ and $H_2 \subset G_2$ be normal subgroups. Prove that $H_1 \times H_2$ is normal in $G_1 \times G_2$ and $(G_1 \times G_2)/(H_1 \times H_2) \simeq (G_1/H_1) \times (G_2/H_2)$.

5. Let a group G be the product of its normal subgroups K and N. Prove that the composition of the inclusion homomorphism $K \to G$ with the canonical homomorphism $G \to G/N$ is an isomorphism.

6. Let K and N be two normal subgroups of a group G such that the composition of the inclusion homomorphism $K \to G$ with the canonical homomorphism $G \to G/N$ is an isomorphism. Prove that $G = K \times N$.

7. Prove that the groups $\mathbb{Z}$ and $\mathbb{Z} \times \mathbb{Z}$ are not isomorphic.

8. Are the groups $(\mathbb{Z}/6\mathbb{Z}) \times (\mathbb{Z}/5\mathbb{Z})$ and $(\mathbb{Z}/3\mathbb{Z}) \times (\mathbb{Z}/10\mathbb{Z})$ isomorphic?

9. Are the groups $(\mathbb{Z}/4\mathbb{Z}) \times (\mathbb{Z}/4\mathbb{Z})$ and $(\mathbb{Z}/8\mathbb{Z}) \times (\mathbb{Z}/2\mathbb{Z})$ isomorphic?

10. Let f and $g : A \to B$ be two homomorphisms of abelian groups written additively. Show that the map $(f + g) : A \to B$ defined by $(f + g)(a) = f(a) + g(a)$ is also a homomorphism. Prove that the set $Hom(A, B)$ of all homomorphisms from A to B is an abelian group with respect to the operation $f + g$.