

HOMEWORK 5

1. Write out the disjoint cycle decomposition of $(1234)(456)(145)$.
2. Determine the order of $(1234)(567)$ in S_7 .
3. Find the largest order of an element in S_5 .
4. Find all elements in S_5 that commute with the cycle (123) .
5. Find all conjugacy classes in S_3 and S_4 .
6. Prove that every element of S_n is a product of transpositions (ij) .
7. Show that S_3 has trivial center.
8. Prove that there are exactly 6 inner automorphisms of S_3 .
9. Prove that every automorphism of S_3 is inner. (Hint: Show that there are at most 6 automorphisms of S_3 by looking at their action on elements of order 1, 2 and 3.)
10. Let $N = \{e, (12)(34), (13)(24), (14)(23)\} \subset S_4$. Show that N is a normal subgroup of S_4 .