

HOMEWORK 1

1. Is the set of positive real numbers with the operation $a * b = a^b$ a group?
2. Let $G = \{e, a, b\}$ be a group of three elements with the identity e . Prove that $ab = e$ and $a^2 = b$.
3. Show that $((ab)c)d = a(b(cd))$ for all elements a, b, c, d in a group.
4. Prove that for an element a in a group, $(a^{-1})^n = (a^n)^{-1}$.
5. Prove that for an element a in a group, $a^n \cdot a^m = a^{n+m}$ for every $n, m \in \mathbb{Z}$.
6. Let G be an abelian group and let $a, b \in G$ be elements of order 2 and 3 respectively. Show that ab has order 6.
7. Find the number of elements of order 3 in a cyclic group of order 12.
8. Find all generators in the group $\mathbb{Z}/18\mathbb{Z}$.
9. Show that if G is a group in which $(ab)^2 = a^2b^2$ for all $a, b \in G$, then G is abelian.
10. Prove that if G is a finite group of even order, then G contains an element of order 2. (Hint: Consider the set of pairs (a, a^{-1}) .)