Class Information for

Foundations of Number Theory

Math 435, Fall 2006

MWF 2-2:50pm, 303 Addams Hall

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Office hours: Monday 3-4pm, Wednesday 3-4pm, Friday 11am-12.

Course text: *Elementary Number Theory in Nine Chapters,* Second Edition (Paperback) by James J. Tattersall, Cambridge University Press, 2005, ISBN 0521615240. An electronic version of the first edition of this book is accessible at

http://proxy.cc.uic.edu/login?url=http://site.ebrary.com/lib/uic/Doc?id=10073533

Syllabus: *Mathematics is the queen of the sciences and number theory is the queen of mathematics.* (Carl Friedrich Gauss, as quoted in: Sartorius von Walterhausen, *Gauss zum Gedächtniss,* Leipzig, 1856, p.79.)

Number theory is one of the oldest branches of pure mathematics, and one of the largest; its aim is the study of the integers. It is a subject abundant with problems which are easily stated, but whose solution is often extremely difficult and sometimes requires sophisticated methods from other branches of mathematics. Examples include:

- Catalan's conjecture (formulated 1844, and proved by Mihăilescu only in 2002): 8 and 9 are the only successive integer powers.
- Fermat's last theorem (stated in 1637, but not proved until 1994, by Andrew Wiles): there are no nonzero integers x, y, z such that $x^n + y^n = z^n$ for some integer $n > 2$.

Many other ones are still open, for example:

- The twin prime conjecture: there are infinitely many prime pairs, such as $(3, 5), (5, 7), (11, 13), \ldots$
- The Goldbach conjecture: every even number > 2 is the sum of two primes, e.g., $4 = 2 + 2$, $6 = 3 + 3$, $8 = 3 + 5$, $10 = 5 + 5$, $\ldots$

Sometimes the answer turns out not to be what someone guessed (centuries ago): In 1769, while thinking about Fermat's last theorem, Leonhard Euler conjectured that the equation $a^4 + b^4 + c^4 = d^4$ would also have no nonzero integer solutions. In 1998, Noam Elkies of Harvard University found the first counterexample: the equation is true when $a = 2,682,440$, $b = 15,365,639$, $c = 18,796,760$, and $d = 20,615,673$.

In this course, we will focus on "elementary number theory": the division algorithm, the Euclidean algorithm (existence of greatest common divisors), elementary properties of primes (unique factorization, the infinitude of primes), congruences, including Fermat's little theorem, quadratic residues, and so forth. The term "elementary" is used in this context only to suggest that no advanced tools from other areas are used—not that the results themselves are simple. Indeed, many of the results which we will discuss (e,g., the Quadratic Reciprocity law, the Chinese Remainder Theorem) turned out, in retrospect, to presage more sophisticated tools and themes introduced later in history.

Homework: One of the main goals of this course, in my eyes, is to teach you how to read mathematical texts, and how to formulate clear mathematical arguments yourself. Since this is best done without time pressure, homeworks provide the ideal medium to practice your proof-writing skills. Therefore: Homework will be assigned every two weeks or so, handed out in class and due at the beginning of class. Up to 3 individuals may work together on homework problems (and I encourage you to do so), but when you turn in the problem set you should acknowledge that you have collaborated.

Put the following information in the upper right hand corner of the first page:

Your Name(s)

Math 435, Homework # x

On each additional page, put your name(s) in the upper right-hand corner. Work single-sided, that is, write on only one side of each sheet of paper. STAPLE any homework that is more than one page long. Remove all perforation before submitting. Write legibly.

No late homework will be accepted.

Homework that fails to meet the above requirements will be marked "Unacceptable" and returned unread.

Exams: There will be three examinations, in class, on *Friday, September 29, Friday, November 3,* and on *Friday, December 8.* (There will be no final exam.)

Students with examinations which conflict with the exams in this course are responsible for discussing makeup examinations with me no later than two weeks prior to the exam in question.

No books, calculators, scratch paper or notes will be allowed during exams. Students are expected to be thoroughly familiar with the University's policy on academic integrity. The University has instituted serious penalties for academic dishonesty.

Copying work to be submitted for grade, or allowing your work to be submitted for grade to be copied, is considered academic dishonesty.

It is University policy that students with disabilities who require accommodations for access and participation in this course must be registered with the Office of Disability Services.

Grading policy: Homework: 25%. Exams: 25% each.

Letter grades: Roughly computed as follows: A = 90–100%, B = 80–89%, C = 70–79%, D = 60–69%, E = below 60%.