Formal Logic

Math 430, Fall 2004

Rules of the Sequent Calculus

Antecedent Rule
$$\frac{\Gamma \quad \varphi}{\Gamma' \quad \varphi} \text{ if } \Gamma \subseteq \Gamma' \qquad (1)$$

Assumption Rule
$$\frac{}{\Gamma \quad \varphi} \text{ if } \varphi \in \Gamma \qquad (2)$$

Proof by Cases Rule
$$\frac{\begin{array}{lll}\Gamma & \psi & \varphi \\ \Gamma & \neg\psi & \varphi\end{array}}{\begin{array}{ll}\Gamma & \varphi\end{array}} \qquad (3)$$

Contradiction Rule
$$\frac{\begin{array}{lll}\Gamma & \neg\varphi & \psi \\ \Gamma & \neg\varphi & \neg\psi\end{array}}{\begin{array}{ll}\Gamma & \varphi\end{array}} \qquad (4)$$

Disjunction Rule for the Antecedent
$$\frac{\begin{array}{lll}\Gamma & \varphi & \theta \\ \Gamma & \psi & \theta\end{array}}{\begin{array}{lll}\Gamma & (\varphi \vee \psi) & \theta\end{array}} \qquad (5)$$

Disjunction Rules for the Consequent
$$\frac{\Gamma \quad \varphi}{\Gamma \quad (\varphi \vee \psi)} \qquad (6.a)$$
$$\frac{\Gamma \quad \varphi}{\Gamma \quad (\psi \vee \varphi)} \qquad (6.b)$$

Reflexivity of Equality Rule
$$\frac{}{t = t} \qquad (7)$$

Substitution for Equality Rule
$$\frac{\Gamma \quad \varphi\frac{t}{v}}{\Gamma \quad t = t' \quad \varphi\frac{t'}{v}} \qquad (8)$$

$\exists$-introduction in the Consequent Rule
$$\frac{\Gamma \quad \varphi\frac{t}{v}}{\Gamma \quad \exists v\varphi} \qquad (9)$$

$\exists$-introduction in the Antecedent Rule
$$\frac{\Gamma \quad \varphi\frac{w}{v} \quad \psi}{\Gamma \quad \exists v\varphi \quad \psi} \qquad (10)$$
if w not free in Γ, $\exists v\varphi$, ψ

Some Derived Rules

Second Contradiction Rule

$$\dfrac{\Gamma \quad \psi \qquad \Gamma \quad \neg\psi}{\Gamma \quad \varphi}$$

Chain Rule

$$\dfrac{\Gamma \qquad \varphi \qquad \Gamma \quad \varphi \quad \psi}{\Gamma \qquad \psi}$$

Contrapositive Rules

$$\dfrac{\Gamma \quad \varphi \quad \psi}{\Gamma \quad \neg\psi \quad \neg\varphi}$$

$$\dfrac{\Gamma \quad \neg\varphi \quad \psi}{\Gamma \quad \neg\psi \quad \varphi}$$

$$\dfrac{\Gamma \quad \varphi \quad \neg\psi}{\Gamma \quad \psi \quad \neg\varphi}$$

$$\dfrac{\Gamma \quad \neg\varphi \quad \neg\psi}{\Gamma \quad \psi \quad \varphi}$$

Double Negation Rule

$$\dfrac{\Gamma \quad \neg\neg\varphi}{\Gamma \quad \varphi}$$

Modus Ponens

$$\dfrac{\Gamma \qquad\qquad \varphi \qquad \Gamma \quad (\neg\varphi \vee \psi)}{\Gamma \qquad\qquad \psi}$$

Transitivity of Equality

$$\dfrac{\Gamma \quad t_1 = t_2 \qquad \Gamma \quad t_2 = t_3}{\Gamma \quad t_1 = t_3}$$

Symmetry of Equality

$$\dfrac{\Gamma \quad t_1 = t_2}{\Gamma \quad t_2 = t_1}$$

Substitution of Equals

$$\dfrac{\begin{array}{ll}\Gamma & t_1 = t_1' \\ & \vdots \\ \Gamma & t_n = t_n' \\ \Gamma & Rt_1t_2\cdots t_n\end{array}}{\Gamma \quad Rt_1't_2'\cdots t_n'}$$

$$\dfrac{\begin{array}{ll}\Gamma & t_1 = t_1' \\ & \vdots \\ \Gamma & t_n = t_n'\end{array}}{\Gamma \quad ft_1\cdots t_n = ft_1'\cdots t_n'}$$