Problem Set 2
Due Friday, April 17.

Real Analysis

Math 131A, Spring Quarter 2015

1. Do problem 2.1 in the textbook.
2. Do problems 3.3 and 3.4 in the textbook.
3. Do problems 4.1–4.4 for (a), (b), (k), (u), (v), as well as 4.14, in the textbook.
4. Let K be a field, i.e., a set equipped with two maps

$$(a,b) \mapsto a+b\colon K\times K \to K, \quad (a,b)\mapsto a\cdot b\colon K\times K\to K$$

   satisfying the axioms (A1)–(A4), (M1)–(M4) and (DL) stated in class.

   (a) Show that the element 0 postulated to exist in (A3) is unique.

   (b) Show that the element 1 postulated to exist in (M3) is unique.

5. Consider the subset $K := \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ of $\mathbb{R}$.

   (a) Show that $0, 1 \in K$, and if $r, s \in K$, then $r + s$ and $r \cdot s$ also belong to K.

   (b) Verify that K equipped with the operations $(r, s) \mapsto r+s$ and $(r, s) \mapsto r \cdot s$ becomes a field.

   (c) Show that there exists a binary relation $\leq$ on K so that K becomes an ordered field. Extra credit: can you find two distinct such relations?